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# PROBABILISTIC VOLTAGE PROFILE EVALUATION IN RADIAL DISTRIBUTION NETWORK USING THREE-POINT ESTIMATION METHOD

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*Abstract:*

**This study presents a probabilistic load flow analysis of a radial distribution network using the Three-Point Estimation Method (3-PEM) to evaluate the impact of load uncertainty on voltage performance. Backward-Forward Sweep was used for the load flow analysis. The analysis was implemented using MATLAB R2022, and the Ayepe 11-kV, 34-bus radial distribution network in Ibadan, Nigeria was adopted as the test system. The results indicate that the minimum bus voltage obtained from deterministic load flow analysis was 0.8609 p.u., while the minimum mean voltage obtained using the 3-PEM was 0.8598 p.u. The maximum voltage standard deviation across the network was 0.0328 p.u., reflecting the extent of voltage variability due to uncertain loading conditions. The findings confirm that voltage violation probability increases with increasing load uncertainty. Notably, buses 6–9 and 26–28, which satisfied voltage limits under deterministic load flow analysis, experienced voltage violations when probabilistic effects were considered. This clearly demonstrates the limitation of deterministic analysis and its tendency to underestimate voltage instability risk in radial distribution networks. Furthermore, a top-10 worst-bus ranking based on voltage violation probability was developed, with buses ordered in descending order of violation risk. Bus 18 exhibited the highest voltage violation probability, while bus 13 recorded the lowest among the critical buses. This ranking provides valuable insight into priority locations for voltage control, reactive power compensation, and reliability enhancement in radial distribution systems.**
*Keywords* —**MATLAB R2022, Three-Point Estimation Method, Voltage, Radial Distribution System, Probability.**

## 1. Introduction

Traditional load flow techniques have historically served as a cornerstone of power system analysis by offering steady-state insights into voltage magnitudes, power flows, and network losses under assumed operating conditions. These classical formulations rely on the premise that system parameters—such as load demand and power generation—are precisely known and remain constant during analysis. While this assumption was acceptable for conventional power systems, it no longer reflects the operational reality of present-day distribution networks, where uncertainty has become a dominant characteristic ([6], [10]). Modern radial distribution systems are increasingly influenced by unpredictable factors, including time-varying consumer demand, intermittent renewable generation, and stochastic prosumer behavior. Such uncertainties undermine the validity of deterministic load flow approaches, which are unable to capture the probabilistic nature of system states under realistic operating conditions. Radial feeders, despite their advantages in terms of simplicity, cost-effectiveness, and protection coordination, are particularly vulnerable to these variations. Even modest fluctuations in load or distributed generation can lead to significant voltage deviations, elevated power losses, or feeder-end voltage instability ([9], [13]). In real distribution networks, electrical demand evolves continuously as a function of daily consumption cycles, climatic conditions, and economic activities. These dynamics necessitate analytical frameworks that move beyond single-point estimates and instead quantify the likelihood of different operating outcomes. Probabilistic load flow (PLF) analysis addresses this requirement by providing statistical descriptions such as expected values, variances, and probability distributions of voltages, currents, and losses, thereby offering a more realistic assessment of network performance under uncertainty [2]. Probabilistic Load Flow methodologies model uncertain inputs, including loads and distributed energy resources, as random variables characterized by appropriate probability density functions. Consequently, system responses are also expressed probabilistically. Although Monte Carlo Simulation (MCS) is widely recognized for its robustness and accuracy in probabilistic analysis, its reliance on thousands of repeated load flow calculations results in high computational cost. This limitation restricts its suitability for large-scale networks and time-constrained planning or operational studies [16].

To reduce computational burden while maintaining acceptable accuracy, alternative approximation techniques such as the Point Estimation Method (PEM) have been proposed. The three-point estimation method (3-PEM) represents an enhancement over earlier two-point schemes by incorporating higher-order statistical information, including skewness effects. In this approach, each uncertain input variable is replaced by three strategically selected concentration points with corresponding weighting coefficients derived from its statistical moments. This structure enables efficient estimation of output distributions using a limited number of deterministic load flow runs ([7], [14]). The integration of 3-PEM with backward–forward sweep load flow algorithms commonly employed for radial distribution networks offers a practical and computationally efficient framework for probabilistic assessment. Through this hybrid approach, key performance indices such as voltage profiles, feeder losses, and voltage violation probabilities can be evaluated under uncertainty without excessive computational effort. As a result, 3-PEM-based PLF is well suited for distribution system planning, reliability evaluation, and voltage stability analysis in networks with high uncertainty penetration ([5], [1]).

## 2. *Literature Review*

Probabilistic load flow analysis represents a significant departure from traditional deterministic power flow formulations by explicitly treating uncertain system inputs such as consumer demand and renewable energy output as stochastic variables. Rather than yielding single-point operating solutions, this approach describes network responses, including bus voltages, line flows, and system losses, in statistical terms. As a result, probabilistic load flow provides system planners and operators with a more realistic assessment of network behavior under variable and uncertain conditions. Although Monte Carlo Simulation is widely used as a reference technique for stochastic analysis, its high computational cost has driven interest in faster approximation methods, particularly those based on point estimation.

Within this class of approximation techniques, the three-point estimation method (3PEM) has received considerable attention due to its ability to achieve acceptable accuracy with a limited number of deterministic load flow executions. By strategically selecting representative concentration points for each random input, 3PEM is able to estimate multiple statistical characteristics of system outputs up to the fourth moment without the need for extensive sampling.

Recent research has focused on refining and extending point estimation–based probabilistic load flow methods. For example, [23] introduced an enhanced 3PEM formulation that employs Halton sequence sampling to improve the uniformity of input representation, thereby increasing accuracy while avoiding reliance on higher-order statistical moments. Their results demonstrated superior performance on standard IEEE benchmark networks compared to conventional point estimation approaches [23]. In a related study, Singh, Moger, and Jena (2025) proposed a modified point estimation framework tailored for wind-integrated systems, showing that improved treatment of distribution tails leads to more reliable characterization of extreme operating conditions critical for reliability studies [22].

In parallel, alternative probabilistic load flow strategies—particularly analytical and non-parametric methods—are gaining momentum. [21] presented a non-parametric probabilistic framework that combines adaptive kernel density estimation with Latin Hypercube Sampling, enabling accurate modeling of arbitrary probability distributions associated with photovoltaic and wind generation. Their approach achieved accuracy comparable to Monte Carlo Simulation while significantly reducing computational effort [21]. Although these methods are not specific to 3PEM, they highlight the broader movement toward efficient stochastic analysis techniques in power system studies.

Despite notable methodological progress, a significant portion of the literature continues to rely on IEEE benchmark or synthetic test systems, limiting insight into real-world operating environments. Moreover, the coupling of point estimation–based probabilistic load flow with optimization frameworks for distributed generation planning has demonstrated the usefulness of stochastic analysis in supporting both technical performance evaluation and economic decision-making in network design [24]. Comprehensive review studies further emphasize the inherent trade-offs among Monte Carlo, analytical, and point estimation approaches, recommending method selection based on system scale, uncertainty characteristics, and computational constraints [20].

Notwithstanding the growing body of work on 3PEM-based probabilistic load flow, empirical studies focusing on real distribution systems in developing power networks remain limited. In particular, Nigerian radial distribution networks exhibit distinct features including weak grid strength, highly variable load patterns, and pronounced solar and wind intermittency that differ markedly from those of benchmark test feeders. To address this gap, the present study applies probabilistic load flow analysis using the three-point estimation method to a Nigerian 34-bus radial distribution network located in Ibadan, with the objective of assessing the suitability of 3PEM under realistic local operating conditions.

## 3. *Materials and Method*

The study presents the probabilistic load flow analysis of the 11 kV AYEPE 34-bus radial distribution network of the Ibadan Electricity Distribution Company (IBEDC) using the Three-Point Estimation Method (3-PEM)**.** The approach accounts for uncertainties in load demand while preserving the radial topology and operational characteristics of the AYEPE feeder. The proposed method enables efficient estimation of voltage profiles and voltage violation probabilities with significantly reduced computational effort.

### *3.1 Description of the AYEPE 34-Bus IBEDC Network*

The AYEPE distribution network is a single-source, radial 11 kV feeder consisting of 34 buses and 33 distribution lines**.** Bus 1 represents the 11 kV injection substation, while buses 2–34 serve as load buses. The network is characterized by high R/X ratios typical of Nigerian distribution feeders. The test system carries an aggregate real power demand of 3.715 MW and a total reactive power demand of 2.3 Mvar. The load flow analysis was carried out using the Backward Forward Sweep algorithm.

### *3.2 Deterministic Load Flow Model of AYEPE Network: Bus Power Injection Model*

The complex power demand at bus $i$ is expressed as:

$$S_i = P_i + jQ_{i,} \quad i = 2,3, \ldots, 34$$

Where:

- $P_i$ is the real power demand (kW),
- $Q_i$ is the reactive power demand (kVAr).

Bus 1 is treated as the slack (reference) bus:

$$V_1 = 1.0 < 0^o p.u.$$

### *3.3) Branch Current and Voltage Relations:*

The current injected at bus $i$ is:

$$I_i = \frac{(P_i - jQ_i)}{V_i^*}$$

For a branch connecting sending bus $i$ to receiving bus $j$:

$$V_j = V_1 - Z_{ij} I_{ij}$$

Where:

$$Z_{ij} = R_{ij} + jX_{ij}$$

Backward-forward sweep load flow is employed due to the strictly radial structure of the AYEPE feeder.

### *3.4) Stochastic Load Modeling for the AYEPE Feeder:*

Load demand uncertainty is modeled at all load buses (2-34).

$$P_i = \mu_{P_i} + \Delta P_i$$
$$Q_i = \mu_{Q_i} + \Delta Q_i$$

Where:

- $\mu_{P_i}, \mu_{Q_i}$ are the nominal IBEDC load values,
- $\Delta P_i, \Delta Q_i$ represent stochastic deviations.

Each load is assumed to follow a normal distribution:

$$P_i \sim N(\mu_{P_i}, \sigma_{P_i}^2)$$
$$Q_i \sim N(\mu_{Q_i}, \sigma_{Q_i}^2)$$

Typically:

$$\sigma_{P_i} = \beta \mu_{P_i}, \ \sigma_{Q_i} = \beta \mu_{Q_i}$$

Where $\beta \in [0.05, 0.15]$ represents load uncertainty level.

### *3.5) Three-Point Estimation Method (3-PEM) Formulation:*

For each uncertain load variable $X_i \in \{P_i, Q_i\}$ , three concentration points are generated:

$$X_i^{(1)} = \mu_{X_i}$$
$$X_i^{(2)} = \mu_{X_i} + \sqrt{3\sigma_{X_i}}$$
$$X_i^{(3)} = \mu_{X_i} + \sqrt{3\sigma_{X_i}}$$

With associated weights:

$$w_1 = \frac{2}{3}, w_2 = \frac{1}{6}, w_3 = \frac{1}{6}$$

### *3.6) Probabilistic Load Flow Implementation for AYEPE 34-Bus System:*

For each load bus $i$:

1. Other buses are fixed at their mean loads.
2. Three deterministic load flows are executed using $P_i^{(k)}, Q_i^{(k)}$.
3. Output variables are recorded.

Let $V_i^{(k)}$ denote the voltage magnitude at bus $i$ for the $k^{th}$ concentration point.

### *3.7) Mean Bus Voltage:*

$$\mu_{V_i} = \sum_{k=1}^{3} w_k V_i^{(k)}$$

### *3.8) Voltage Variable:*

$$\sigma_{V_i}^2 = \sum_{k=1}^{3} w_k \left(V_i^{(k)}\right)^2 - \mu_{V_i}^2$$

***3.9 Voltage Violation Probability in the AYEPE Network***

IBEDC operational voltage limits are defined as:

$$0.90 \leq |V_i| \leq 1.0\ p.u.$$

The probability of voltage violation at bus $i$ is:

$$P_{viol,i} = \Phi\left(\frac{0.95 - \mu_{V_i}}{\sigma_{V_i}}\right) + 1 - \Phi\left(\frac{1.05 - \mu_{V_i}}{\sigma_{V_i}}\right)$$

## *4.0) Results and Discussion*

### *4.1) Deterministic Load Flow Results:*

A conventional deterministic load flow was first performed using the mean load values. The results provide a baseline for comparison with probabilistic outcomes. The result in figure 2 shows that there was progressive voltage drop along the feeder from bus 1 to bus 18 and a rise in voltage at bus 19 and sudden drop from bus 19 to bus 34. The minimum bus voltage occurring at the farthest bus from the substation is 0.8609pu

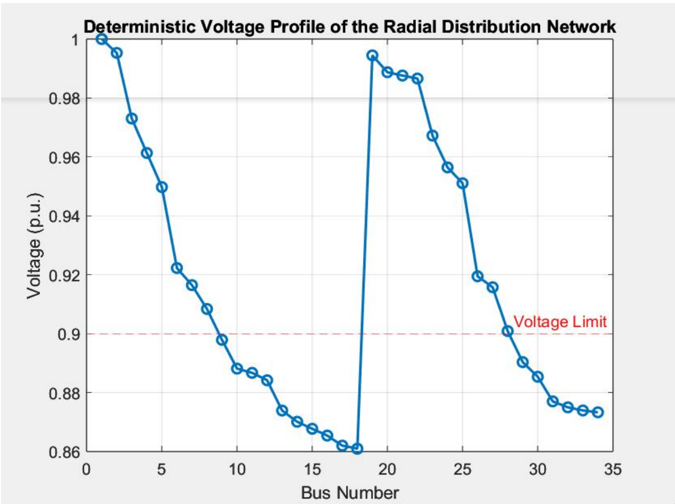


**Fig. 2: Deterministic Voltage Profile of Ayepe 34-Bus Radial Distribution Network**

### *4.2) Probabilistic Voltage Profile Results:*

Using the 3-PEM, the mean and standard deviation of bus voltage magnitudes were computed. The probabilistic voltage profile result in figure 3 reveals that the mean voltage values using 3-PEM closely follows the deterministic voltage profile in figure 2. The minimum mean voltage (3-PEM) is 0.8598 p.u. Figure 4 shows voltage standard deviation using 3-PEM. It shows the voltage standard deviation increases with distance from the substation. The maximum voltage Standard deviation is 0.0328 p.u. The end buses exhibit the highest voltage variability due to cumulative load uncertainty and they are far away from the substation. Figure 4 shows a comparison of deterministic voltage profile and the probabilistic voltage profile.

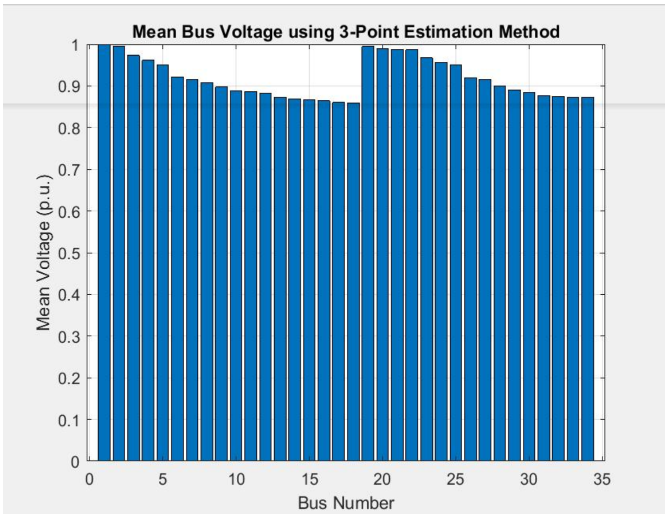


**Fig. 3: Mean Bus Voltage Profile Using 3-PEM**

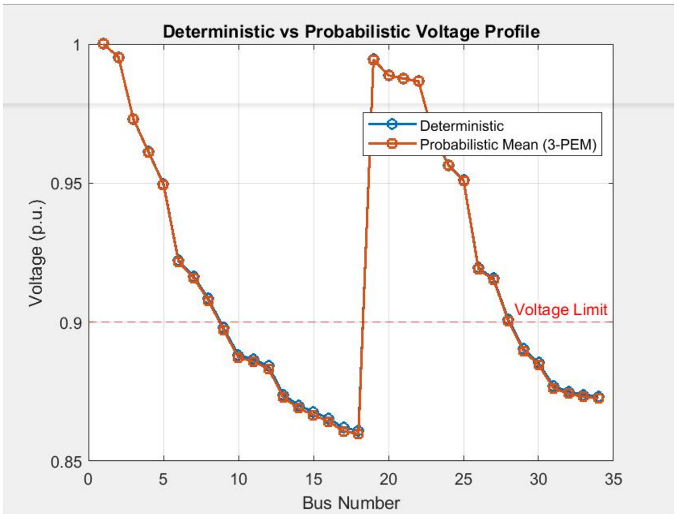


**Fig. 4: Deterministic vs Probabilistic load flow analysis comparison**

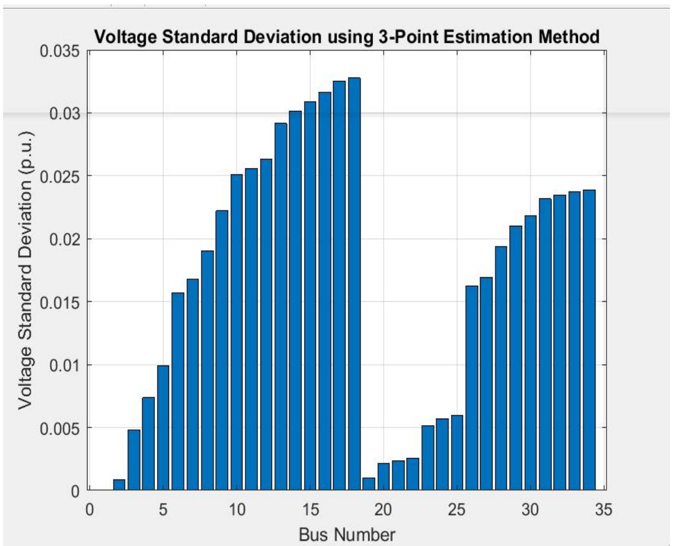

**Fig. 5: Standard Deviation of Bus Voltages Using 3-PEM)**

### *4.3) Probability of Voltage Violation:*

The probability of voltage violation was evaluated by estimating the likelihood of bus voltages dropping below the acceptable minimum limit of 0.90 p.u. As shown in Fig. 6, buses 1–5 and 19–25 did not satisfy the voltage violation condition $P(V<0.90)>5\%$, owing to their closeness to the substation and relatively lighter load levels. Conversely, buses 6–18 and 26–34, which are located toward the feeder extremities, exhibited higher probabilities of undervoltage due to increased loading and demand variability. The results further confirm that voltage violation probability increases with load uncertainty. Notably, buses 6–9 and 26–28 maintained acceptable voltage levels under deterministic load flow conditions (Fig. 2) but experienced voltage violations when probabilistic load flow was considered. This highlights the limitation of deterministic analysis and underscores its tendency to underestimate voltage instability risk in radial distribution networks.

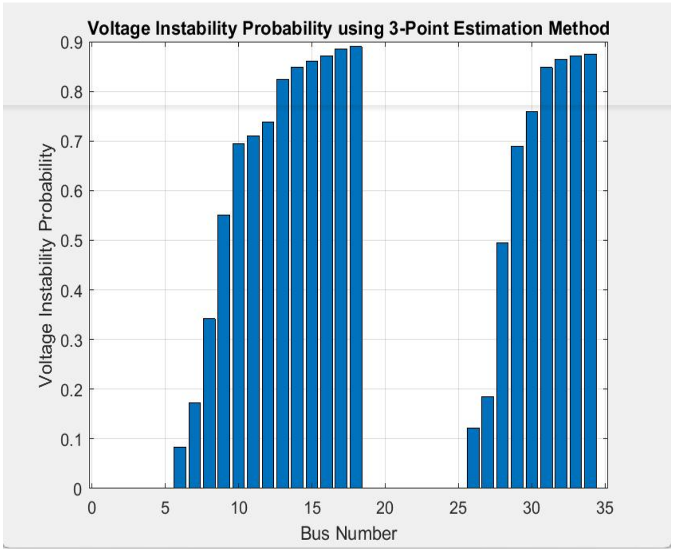


**Fig. 6: Probability of Voltage Violation Across Buses**

### *4.4) Worst-Bus Ranking Based on Voltage Violation Probability:*

Table 1 and fig. 7, illustrates the top-10 worst-bus ranking derived from voltage violation probability, with buses ordered in descending magnitude of violation risk with bus 18 having the highest voltage violation probability value and bus 13 having the lowest voltage violation probability value. This ranking according to voltage violation probability of each critical bus, identifies the priority locations for voltage support and reactive power compensation. For each bus, the table reports the corresponding voltage violation probability alongside the deterministic voltage value, thereby providing contextual insight into the disparity between deterministic and probabilistic voltage assessments.

**Table 1: Top-10 Worst-Bus Ranking Based on Voltage Violation Probability**

| Bus Number | Voltage Violation Probability | Deterministic Voltage |
|---|---|---|
| 18 | 0.89019 | 0.86095 |
| 17 | 0.8859 | 0.86197 |
| 34 | 0.87525 | 0.87391 |
| 33 | 0.87103 | 0.87391 |
| 16 | 0.87094 | 0.86539 |
| 32 | 0.86376 | 0.87498 |
| 15 | 0.86021 | 0.86769 |
| 14 | 0.84819 | 0.87007 |
| 31 | 0.84749 | 0.87698 |
| 13 | 0.824 | 0.87389 |

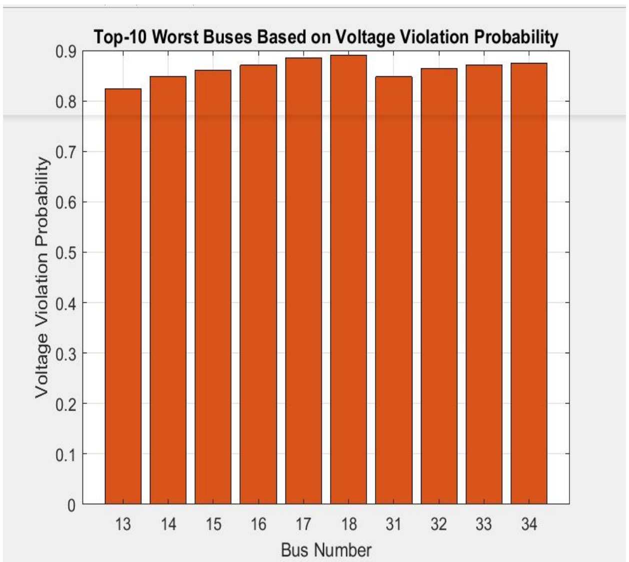


**Fig. 7: Top-10 Worst Buses Based on Voltage Violation Probability**

## *5) Conclusion:*

The results demonstrate that the Three-Point Estimation Method effectively captures the impact of load uncertainty on system performance with significantly reduced computational burden compared to Monte Carlo simulation. The close agreement between deterministic and probabilistic mean values validates the accuracy of the method, while the additional information on variance and violation probability provides deeper insight into system reliability.

For Nigerian radial distribution networks, where load demand is highly uncertain due to irregular consumption patterns and embedded generation, probabilistic load flow analysis using 3-PEM offers a practical and efficient tool for planning and operational studies.

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