OPTIMIZATION OF STRENGTH AND DURABILITY PROPERTIES OF ABUJA LATERITE STABILIZED WITH ORDINARY PORTLAND CEMENT AND RICE-HUSK ASH FOR STRUCTURAL EARTH BLOCK PRODUCTION

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Abstract: *This study investigates the optimization of Ordinary Portland Cement (OPC)–Rice Husk Ash (RHA) stabilized lateritic soil for the production of sustainable structural earth blocks. The research was designed to evaluate the pozzolanic activity of RHA, examine the consolidation and compressive strength characteristics of the stabilized soil blends, identify optimal mix ratios, and provide recommendations for practical applications in sustainable construction. Laboratory experiments, supported by statistical and computational modelling using the* NonLinearModelFit *function in the Wolfram Language, were conducted on thirty-five (35) mixture formulations based on Scheffé's optimization algorithm. Chemical analysis revealed that RHA contained 70.48%* SiO_2*, confirming its status as a highly reactive pozzolan suitable for cementitious applications. The inclusion of RHA and OPC significantly improved the consolidation properties of the lateritic soil, as reflected by reduced coefficients of compressibility and enhanced stiffness. The compressive strength tests indicated that moderate RHA replacement (approximately 15%) combined with 30% OPC yielded optimal performance, achieving strengths in the range of 3.7–4.6 MPa suitable for structural block applications. The derived nonlinear mixture models achieved high predictive accuracy (*R^2 *= 97.8–99.4%), validating their reliability for mix proportion optimization. The study concludes that the synergistic use of OPC and RHA effectively enhances the engineering properties of lateritic soil while promoting sustainable, low-carbon construction. Consequently, the optimized cement–RHA–laterite blend is recommended as an eco-efficient material for the production of durable and affordable structural earth blocks in tropical environments.*

Keywords: Lateritic soil; Rice Husk Ash; Ordinary Portland Cement; Soil stabilization; Structural earth blocks; Mixture optimization; Sustainable construction.

1. Introduction

The demand for affordable and sustainable housing materials in Nigeria continues to rise due to rapid urbanization, population growth, and economic pressures. Conventional construction materials such as reinforced concrete and fired clay bricks are often associated with high embodied energy and increased construction costs, making them less accessible for low- and middle-income housing development. As a result, attention has increasingly shifted toward locally available materials such as lateritic soils for sustainable construction applications.

Lateritic soil is abundantly available in tropical regions and has historically been used for earth construction. However, its engineering performance in natural form is often inadequate for structural applications due to variability in mineral composition, limited strength, and susceptibility to volume change under load (Das & Sobhan, 2014; Terzaghi, 1943). Stabilization using Ordinary Portland Cement (OPC) has therefore become a common practice to enhance its strength and durability (Osinubi, 1998). While cement stabilization improves compressive strength and stiffness, excessive cement usage increases cost and carbon emissions, thereby reducing sustainability.

To address these concerns, agricultural by-products such as Rice Husk Ash (RHA) have been investigated as partial cement replacement materials. RHA is rich in amorphous silica and exhibits strong pozzolanic properties when properly processed (Alhassan, 2008; Mehta & Monteiro, 2014). When combined with cement, RHA reacts with calcium hydroxide to form additional calcium silicate hydrate (C–S–H), improving microstructural densification and long-term strength development. Several studies have reported improved geotechnical properties of lateritic soils stabilized with RHA–cement blends (Osinubi & Eberemu, 2010; Akinmusuru, 2011). However, many of these studies rely on trial-and-error mix proportions without systematic optimization. Mixture experiment methodology provides a scientific framework for optimizing multicomponent systems. Unlike factorial designs, mixture designs consider that component proportions are interdependent and must sum to unity (Scheffé, 1958, 1963). Scheffé's simplex lattice approach enables the development of predictive polynomial models that describe the response surface within the mixture domain. Such models allow simultaneous evaluation of multiple performance parameters and identification of optimal compositions (Montgomery, 2013; Myers et al., 2009). Despite the growing application of RHA in soil stabilization, limited research has applied higher-degree Scheffé mixture models to optimize both compressive strength and consolidation characteristics of stabilized lateritic soils, particularly within the Abuja region. Furthermore, few studies have developed predictive mathematical models capable of estimating geotechnical performance within the mixture space. This study therefore applies Scheffé's fourth-degree simplex lattice model to optimize the proportions of lateritic soil, cement, rice husk ash, and water for structural earth block production. The study evaluates compressive strength, coefficient of consolidation (Cv), and coefficient of volume compressibility (Mv), and develops validated predictive models to guide sustainable mixture design.

2. Materials and Methods

2.1 Materials

Three primary materials were used in this study: lateritic soil, Rice Husk Ash (RHA), and Ordinary Portland Cement (OPC). The lateritic soil was collected from the Julius Berger road construction site along Wole Soyinka Way, Abuja, Nigeria. The selection of Abuja laterite was based on its widespread application in local construction and the need for location-specific stabilization assessment. The collected samples were air-dried and sieved through a 200 µm sieve to remove organic impurities and oversized particles.

Rice Husk Ash (RHA) was obtained from Onyx Rice Mill, Bida, Niger State. The ash was air-dried and sieved to ensure fine particle consistency suitable for pozzolanic reactions. Ordinary Portland Cement (OPC) was sourced from a building materials market within the Federal Capital Territory, Abuja.

2.2 Sample Preparation

Upon collection, laterite and RHA samples were air-dried at room temperature to achieve stable moisture conditions. The materials were sieved through a 200 µm sieve to ensure uniform particle size distribution. This preparation minimized variability and ensured consistency in subsequent laboratory testing.

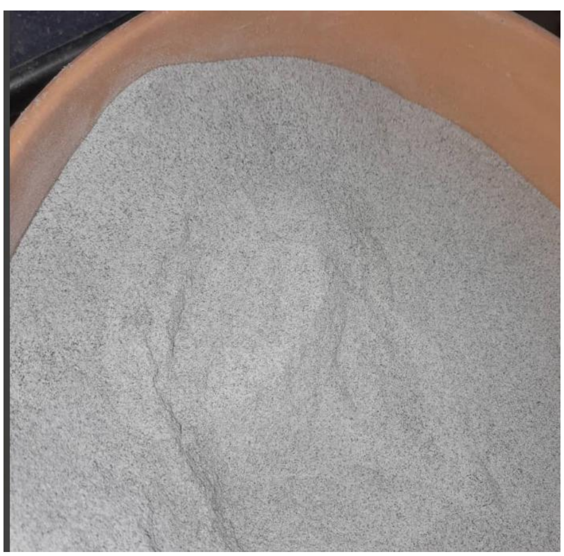


Fig. 1: Sample Rice Husk Ash (RHA)

Fig. 2: Laterite Sample Before Sieving

Fig. 3: Sample Laterite, RHA sent for XRF Testing

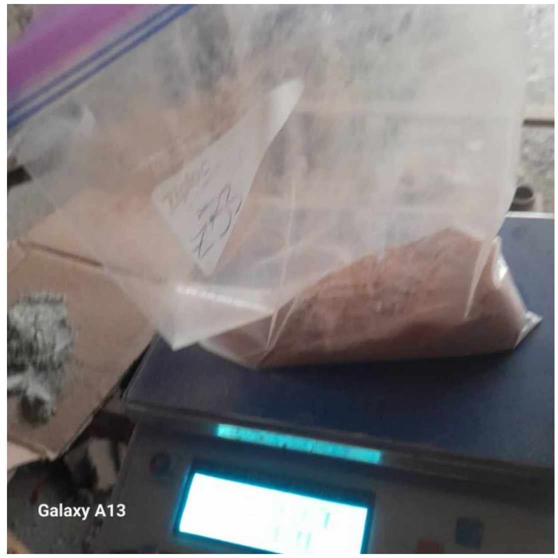


Fig. 4: Weighing Sample Laterite sent for XRF Testing

2.3 Soil Characterization

2.3.1 Atterberg Limits

Atterberg limits tests were conducted on the natural laterite in accordance with BS 1377-2:1990 specifications. The liquid limit, plastic limit, and plasticity index were determined to evaluate the soil's consistency and plastic behaviour.

2.3.2 Particle Size Distribution

Particle size distribution was determined using sieve analysis in accordance with ASTM D6913-04 (2009) E1. The gradation characteristics were evaluated to assess the suitability of the laterite for stabilized earth block production.

2.3.3 Consolidation Test

Consolidation tests were performed on stabilized mixtures in accordance with EN ISO 17892-5:2017 (Eurocode7: Geotechnical Design – Part 1: General Rules). The oedometer method was used to determine the coefficient of consolidation (Cv) and the coefficient of compressibility (Mv) from time-settlement and pressure–void ratio relationships.

(Eqn. 1)

$$N(q,m) = \frac{(q+m-1)!}{m!\,(q-1)!}$$

2.4 Mineralogical and Chemical Characterization

Mineralogical characterization of laterite, RHA, and OPC was conducted using X-ray diffraction (XRD) analysis. Approximately 5 g of each anhydrous sample was prepared and analyzed at the Science Laboratory Technology Department, Umaru Musa Yar'adua University, Katsina. The analysis was used to determine oxide composition and confirm the pozzolanic potential of RHA.

2.5 Mix Design and Mixture Optimization

A four-component mixture system consisting of water, Ordinary Portland Cement (OPC), Rice Husk Ash (RHA), and laterite soil was developed for stabilized earth block production.

2.5.1 Water Content

The water content was maintained within 11–15% of the weight of laterite soil to ensure adequate workability and compaction during block production.

2.5.2 Scheffé's Mixture Design

A fourth-degree mixture polynomial based on Scheffé's simplex lattice design was adopted to model the interaction among the four components. The required number of experimental mixture points was determined using the Scheffé polynomial expression:

Where q is the degree of the polynomial and m is the number of components. For this study, $N(4,4)$ a total of 35 experimental trials were generated, consistent with the adopted mixture design.

2.6 Sample Brick Production

Stabilized earth blocks were produced using the computed mixture proportions obtained from the Scheffé design matrix. The materials were thoroughly blended to ensure uniform distribution of OPC and RHA within the laterite matrix. The mixtures were molded, compacted, demoulded, and cured under laboratory conditions prior to testing.

Fig.5: Sample Brick

Fig.6: Sample Brick

2.7 Compressive Strength Testing

Compressive strength tests were conducted on cured specimens using a compression testing machine. The ultimate load at failure was recorded and converted to compressive strength based on specimen dimensions. The compressive strength values served as primary response variables in the optimization model.

Fig.7: Compressive Strength Testing of Sample Brick

Fig.8: Weighing of Sample Brick in Before Compression Test

2.8 Computational Modelling and Statistical Validation

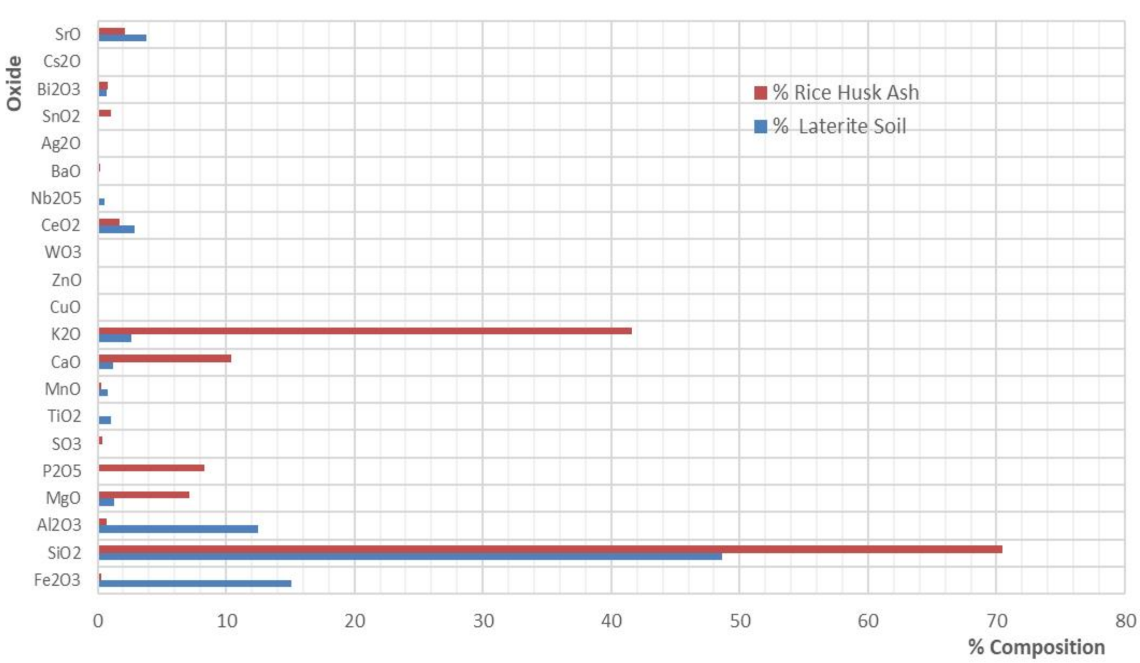
Nonlinear regression modelling was implemented using the NonLinearModelFit function in the Wolfram Language to establish predictive relationships between mixture proportions and performance responses, including compressive strength and consolidation parameters. Model adequacy was evaluated using statistical indicators such as the coefficient of determination (R^2), standard error, t-statistics, and p-values to ensure predictive reliability.

3. Results and Discussion

3.1 Mineralogical Characterization

The mineralogical composition of the laterite and Rice Husk Ash (RHA) is presented in **Figure 9**. The XRD results confirm the high silica content of RHA (approximately 70.48% SiO_2 as reported in the abstract), validating its pozzolanic potential. The predominance of amorphous silica suggests strong reactivity with calcium hydroxide released during cement hydration, leading to secondary calcium silicate hydrate (C–S–H) formation.

Fig. 9: Mineralogical Plot of RHA and Laterite Soil



The laterite exhibited a largely granular mineralogical structure with limited clay fraction, which aligns with the low plasticity behaviour observed in subsequent Atterberg limit tests. The mineralogical interaction between OPC and RHA therefore provides the chemical basis for the observed improvements in consolidation and compressive strength performance.

3.2 Atterberg Limits of Natural Laterite

The liquid limit and plastic limit test results are presented in **Table 1** and **Table 2**, respectively. The soil exhibited a liquid limit (LL) of 22.4%, plastic limit (PL) of 22.8%, and a very low plasticity index (PI) of 0.42%.

The extremely low PI indicates non-plastic to slightly plastic behaviour, suggesting minimal clay activity and a predominantly granular composition. This classification supports the suitability of the soil for stabilization, as low-plastic soils respond effectively to cementitious binding. The low shrinkage potential further enhances its desirability for structural earth block production, reducing susceptibility to cracking during drying and curing.

Table 1: Liquid Limits Test Results of laterite soil sample

S/No.	Sample Tag	wt. of Dish (g)	wt. of Dish + wet Sample	wt. of wet Sample (g)	wt. of Dish + dry Sample (g)	wt. of dry Sample (g)	Moisture Content	% Moisture Content	Penetration (mm)
1	C2	77.6	83.2	5.60	82.0	4.40	1.2	27.273	18.47
2	B1	75.2	82.3	7.10	81.0	5.80	1.3	22.414	25.88
3	E2	75.21	83.6	8.39	82.4	7.19	1.2	16.690	32.51
4	C1	76.2	85.1	8.90	84.0	7.80	1.1	14.103	35.39

Table 2: Plastic Limits Test Results of laterite soil sample

S/No.	Sample Tag	wt. of Dish (g)	wt. of Dish + wet Sample	wt. of wet Sample (g)	wt. of Dish + dry Sample (g)	wt. of dry Sample (g)	Moisture Content	% Moisture Content

1	A1	80.0	82.0	2	81.6	1.6	0.4	25.00
2	E1	78.2	79.7	1.5	79.4	1.2	0.3	25.00
3	A2	80.8	82.4	1.6	82.1	1.3	0.3	23.08
4	A3	76.1	77.4	1.3	77.2	1.1	0.2	18.18

3.3 Consolidation Behaviour of OPC–RHA Stabilized Laterite

The consolidation and compressibility response of the stabilized mixtures are presented in Table 3. The incorporation of OPC and RHA significantly influenced the coefficient of consolidation (Cv) and coefficient of compressibility (Mv). Blends such as A4 and B34 demonstrated reduced compressibility and enhanced stiffness relative to other mixtures. The reduction in compressibility is attributed to the formation of cementitious bonds and pore refinement due to pozzolanic reactions. The densified matrix structure restricts particle rearrangement under applied load, thereby improving settlement resistance.

Table 3: Consolidation, and Compressive Strength Test of stabilized Laterite Blocks

S/No.	**Points**	**Consolidation Tests**								**Compressive Strength**			
		Coefficient of Consolidation (m^2/yr)				**Coefficient of Soil Compressibility (m^2/MN)**				**Compressive Strength (N/mm^2) @ 28 days**			
		Test 1	**Test 2**	**Stan. Dev.**	**Mean %**	**Test 1**	**Test 2**	**Stan. Dev.**	**Mean %**	**Test 1**	**Test 2**	**Stan. Dev.**	**Mean (MPa)**
1	**A1**	6.274	5.784	0.3464	6.029	0.199	0.322	0.0871	0.260	4.776	4.466	0.2188	4.621
2	**A2**	6.156	5.888	0.1900	6.022	0.331	0.363	0.0225	0.347	2.141	2.577	0.3081	2.359
3	**A3**	5.766	5.501	0.1871	5.633	0.655	0.523	0.0936	0.589	4.181	3.005	0.8315	3.593
4	**A4**	5.468	5.229	0.1688	5.348	0.770	0.456	0.2219	0.613	5.649	2.131	2.4873	3.890
5	**B12**	6.212	5.624	0.4151	5.918	0.240	0.318	0.0555	0.279	3.113	3.717	0.4273	3.415
6	**B13**	6.151	5.845	0.2168	5.998	0.340	0.389	0.0349	0.364	2.183	2.747	0.3989	2.465
7	**B14**	5.934	5.632	0.2140	5.783	0.547	0.486	0.0432	0.516	3.391	3.170	0.1559	3.280
8	**B23**	5.947	5.651	0.2090	5.799	0.536	0.469	0.0473	0.503	3.270	3.083	0.1326	3.176
9	**B24**	5.766	5.501	0.1871	5.633	0.655	0.523	0.0936	0.589	4.181	3.005	0.8315	3.593
10	**B34**	5.608	5.363	0.1731	5.486	0.723	0.502	0.1562	0.612	4.925	2.537	1.6887	3.731
11	**G12**	6.212	5.624	0.4151	5.918	0.240	0.318	0.0555	0.279	3.113	3.717	0.4273	3.415
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17	**D34**	5.608	5.363	0.1731	5.486	0.723	0.502	0.1562	0.612	4.925	2.537	1.6887	3.731
18	**D23**	5.947	5.651	0.2090	5.799	0.536	0.469	0.0473	0.503	3.270	3.083	0.1326	3.176
19	**D23**	5.947	5.651	0.2090	5.799	0.536	0.469	0.0473	0.503	3.270	3.083	0.1326	3.176
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21	**D13**	6.151	5.845	0.2168	5.998	0.340	0.389	0.0349	0.364	2.183	2.747	0.3989	2.465
22	**D14**	5.934	5.632	0.2140	5.783	0.547	0.486	0.0432	0.516	3.391	3.170	0.1559	3.280
23	**E1123**	6.175	5.681	0.3494	5.928	0.298	0.387	0.0624	0.343	2.181	2.913	0.5177	2.547
24	**E1124**	6.151	5.845	0.2168	5.998	0.340	0.389	0.0349	0.364	2.183	2.747	0.3989	2.465
25	**E1134**	6.042	5.752	0.2053	5.897	0.455	0.436	0.0139	0.445	2.822	2.959	0.0963	2.890
26	**E2234**	5.852	5.563	0.2045	5.707	0.604	0.507	0.0689	0.556	3.759	3.169	0.4169	3.464
27	**Z1123**	6.175	5.681	0.3494	5.928	0.298	0.387	0.0624	0.343	2.181	2.913	0.5177	2.547
28	**Z1124**	6.151	5.845	0.2168	5.998	0.340	0.389	0.0349	0.364	2.183	2.747	0.3989	2.465
29	**Z1134**	6.042	5.752	0.2053	5.897	0.455	0.436	0.0139	0.445	2.822	2.959	0.0963	2.890
30	**Z2234**	5.852	5.563	0.2045	5.707	0.604	0.507	0.0689	0.556	3.759	3.169	0.4169	3.464
31	**N1123**	6.175	5.681	0.3494	5.928	0.298	0.387	0.0624	0.343	2.181	2.913	0.5177	2.547
32	**N1124**	6.151	5.845	0.2168	5.998	0.340	0.389	0.0349	0.364	2.183	2.747	0.3989	2.465
33	**N1134**	6.042	5.752	0.2053	5.897	0.455	0.436	0.0139	0.445	2.822	2.959	0.0963	2.890
34	**N2234**	5.852	5.563	0.2045	5.707	0.604	0.507	0.0689	0.556	3.759	3.169	0.4169	3.464
35	**Z1234**	5.941	5.641	0.2118	5.791	0.542	0.478	0.0452	0.510	3.330	3.137	0.1365	3.234

The optimal mixture configurations for consolidation responses are presented in Table 4. The computational results identified pseudo-component proportions that minimize compressibility while maintaining structural integrity. These findings confirm that partial replacement of OPC with RHA can enhance stiffness without excessive cement consumption, contributing to both economic and environmental efficiency.

Table 4: Optimal mixture configurations for corresponding Consolidation responses

S/No.	**Component**	**Optimal Coefficient of Consolidation**								
		Xi	**Mix-1**	**Mix-2**	**Mix-3**	**Mix-4**	**Mix-5**	**Mix-6**	**Mix-7**	**Mix-8**
1	Water	0.77198	0.301	0.305	0.334	0.329	0.341	0.313	0.318	0.318
2	Ordinary Portland Cement	0.05779	0.151	0.155	0.184	0.179	0.191	0.163	0.168	0.168
3	Ricer Husk Ash	0.01968	0.350	0.361	0.496	0.480	0.530	0.403	0.428	0.428

4	Laterite Soil	0.15055	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500
	Coefficient of Consolidation	1	6.214	5.938	5.654	5.676	5.582	5.978	5.932	5.932

3.4 Compressive Strength Characteristics

The compressive strength results are summarized in **Table 3** The control sample (A1) recorded a baseline strength of 4.621 MPa. Several blends demonstrated improved structural performance. Notably, samples A4 (3.890 MPa) and B34/G34 (3.731 MPa) exhibited competitive strengths while simultaneously showing superior consolidation behaviour. Although slightly lower than the control in some cases, these mixes provided improved stiffness and reduced compressibility, suggesting enhanced long-term performance under sustained loading.

Conversely, mixes such as A2 (2.359 MPa) and B13/G13 (2.465 MPa) underperformed, indicating that excessive RHA content or insufficient cement reduced effective binder formation. This demonstrates the importance of balancing cement hydration and pozzolanic reaction kinetics.

The optimal mixture configuration for compressive strength is presented in **Table 5**, confirming that moderate RHA replacement with sufficient OPC yields optimal strength performance suitable for structural earth block applications.

Table 5: Optimal mixture configurations for corresponding Compressive Strength responses

S/No.	Component	Optimal Mixture Compressive Strength								
		Xi	**Mix-1**	**Mix-2**	**Mix-3**	**Mix-4**	**Mix-5**	**Mix-6**	**Mix-7**	**Mix-8**
1	Water	0.00074	0.343	0.321	0.322	0.340	0.312	0.315	0.317	0.317
2	Ordinary Portland Cement	0.30724	0.193	0.171	0.172	0.190	0.162	0.165	0.167	0.167
3	Ricer Husk Ash	0.15613	0.541	0.437	0.444	0.527	0.390	0.414	0.420	0.420
4	Laterite Soil	0.53590	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500
	Compressive	1	4.049	3.255	3.248	4.151	2.983	2.346	2.900	2.900

Strength

3.5 Computational Mixture Optimization

Scheffé's fourth-degree polynomial model was used to establish predictive relationships between mixture proportions and response variables. The optimal pseudo-component configurations derived for consolidation and compressive strength responses are presented in Eqn(s) 2, 3 and 4. The mixture space representation is illustrated in **Figure 10**, demonstrating the geometric framework of the simplex lattice design. The derived nonlinear models provided predictive capability across the mixture domain.

$$\begin{aligned} X_{cv} = {} & 6.0292X_1 + 6.0220X_2 - 0.2418X_1X_2 - 0.3778X_1^2X_2 + 5.6332X_3 \\ & - 0.0525X_1X_3 + 1.4383X_1^2X_3 - 0.4832X_2X_3 - 0.3496X_1X_2X_3 \\ & + 0.7366X_2^2X_3 + 5.3484X_4 - 0.2980X_1X_4 + 1.3499X_1^2X_4 \\ & - 0.8223X_2X_4 + 5.7461X_1X_2X_4 + 1.2280X_2^2X_4 - 0.06483X_3X_4 \\ & + 0.28694X_1X_3X_4 - 0.19416X_2X_3X_4 + 0.08903X_3^2X_4 \end{aligned} \qquad \text{(Eqn.2)}$$

$$\begin{aligned} X_{Mv} = {} & 0.2603X_1 + 0.3466X_2 - 0.2286X_1X_2 + 0.2614X_1^2X_2 + 0.5889X_3 \\ & + 0.2271X_1X_3 - 0.9366X_1^2X_3 + 0.5980X_2X_3 - 0.06971X_1X_2X_3 \\ & - 0.9153X_2^2X_3 + 0.6131X_4 + 0.65623X_1X_4 - 0.6767X_1^2X_4 \\ & + 1.16731X_2X_4 - 2.66860X_1X_2X_4 - 1.4627X_2^2X_4 + 0.10596X_3X_4 \\ & + 0.01703X_1X_3X_4 + 0.16004X_2X_3X_4 - 0.12358X_3^2X_4 \end{aligned} \qquad \text{(Eqn.3)}$$

$$\begin{aligned} X_f = {} & 4.62095X_1 + 2.3588X_2 - 4.2709X_1X_2 + 7.9450X_1^2X_2 + 3.5926X_3 \\ & + 1.7632X_1X_3 - 16.6644X_1^2X_3 + 8.9959X_2X_3 - 22.3846X_1X_2X_3 \\ & - 16.3875X_2^2X_3 + 3.8900X_4 + 5.7251X_1X_4 - 19.2519X_1^2X_4 \\ & + 16.7295X_2X_4 - 46.8654X_1X_2X_4 - 29.71325X_2^2X_4 \\ & + 1.4341X_3X_4 - 0.8136X_1X_3X_4 + 1.14763X_2X_3X_4 - 2.9532X_3^2X_4 \end{aligned} \qquad \text{(Eqn.4)}$$

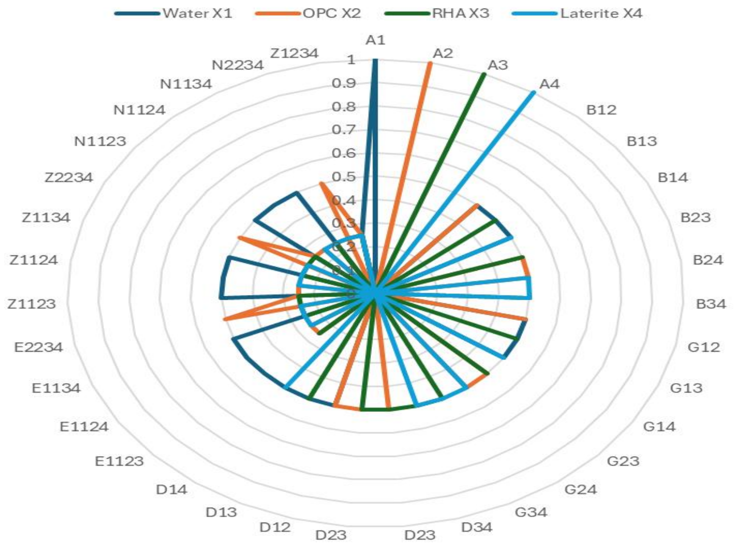

Figure 10: Mixture Space for a $N(4,4)$ Polynomial

$$n(X_i) = \sum_{i=1}^{n} \alpha_i X_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \beta_{i_j} X_i X_j + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \gamma_{i_j} X_i X_j (X_i - X_j) + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \delta_{i_j} X_i X_j (X_i - X_j)^2 + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} \epsilon_{i_{i_{j_k}}} {X_i}^2 X_j X_k + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} \zeta_{i_{i_{j_k}}} X_i {X_j}^2 X_k + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} \eta_{i_{i_{j_k}}} X_i X_j {X_k}^2 + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} \sum_{l=k+1}^{n} \zeta_{i_{j_{k_l}}} X_i X_j X_k X_l \quad \text{(Eqn.5)}$$

$$n(X_i) = X_1\alpha_1 + X_2\alpha_2 + X_3\alpha_3 + X_1X_2\beta_{1_2} + X_1X_3\beta_{1_3} + X_2X_3\beta_{2_3} + X_1^2X_2\gamma_{1_2} - X_1X_2^2\gamma_{1_2} + X_1^2X_3\gamma_{1_3} - X_1X_3^2\gamma_{1_3} + X_2^2X_3\gamma_{2_3} - X_2X_3^2\gamma_{2_3} + X_1^3X_2\delta_{1_2} - 2X_1^2X_2^2\delta_{1_2} + X_1X_2^3\delta_{1_2} + X_1^3X_3\delta_{1_3} - 2X_1^2X_3^2\delta_{1_3} + X_1X_3^3\delta_{1_3} + X_2^3X_3\delta_{2_3} - 2X_2^2X_3^2\delta_{2_3} + X_2X_3^3\delta_{2_3} + X_1^2X_2X_3\epsilon_{1_{1_{2_3}}} + X_1X_2^2X_3\zeta_{1_{1_{2_3}}} + X_1X_2X_3^2\eta_{1_{1_{2_3}}} \quad \text{(Eqn.6)}$$

Also;

$$X_1 + X_2 + X_3 + X_4 = 1 \quad \text{(Eqn.7)}$$

Where characteristics value X are positive real numbers that:

$$0 \le X_i \le 1, i = 1,2,3, \& 4 \quad \text{(Eqn.8)}$$

3.6 Statistical Validation of Optimization Models

The statistical validation of the regression coefficients is presented in Table 6 and Table 7. The extremely low standard errors and statistically significant t-values confirm the robustness of the model parameters.

The coefficient of determination (R^2) ranged between 97.8% and 99.4%, indicating excellent model fit and high predictive accuracy. This validates the reliability of Scheffé's optimization approach for stabilized soil mixture design.

Table 6: Statistical Validation of βi Estimate and Standard Error

		βi Estimate			Standard Error		
S/No.	**βi**	**Coefficient of Consolidation**	**Coefficient of Compressibility**	**Compressive Strength**	**Coefficient of Consolidation**	**Coefficient of Compressibility**	**Compressive Strength**
1	**β1**	6.02927	0.260335	3.50828	7.060E-15	9.577E-16	4.501E-15
2	**β2**	6.0221	0.346668	2.87018	7.060E-15	9.577E-16	4.501E-15
3	**β3**	5.63321	0.588903	1.73712	7.060E-15	9.577E-16	4.501E-15
4	**β4**	5.34846	0.613173	1.36937	7.060E-15	9.577E-16	4.501E-15
5	**β12**	-0.241857	-0.228632	-1.89522	1.073E-12	1.456E-13	6.843E-13
6	**β13**	-0.0525052	0.227153	-0.454149	9.312E-14	1.263E-14	5.937E-14
7	**β14**	-0.298036	0.656238	-0.448341	3.781E-13	5.129E-14	2.411E-13
8	**β23**	-0.483218	0.598094	1.05978	7.205E-13	9.774E-14	4.594E-13
9	**β24**	-0.822301	1.16731	0.900881	8.223E-13	1.115E-13	5.242E-13
10	**β34**	-0.0648367	0.105968	0.0795382	9.736E-14	1.321E-14	6.207E-14
11	**β123**	-0.349654	-0.0697198	4.74186	7.232E-13	9.809E-14	4.611E-13
12	**β124**	5.74612	-2.66861	-3.54054	8.249E-13	1.119E-13	5.259E-13
13	**β134**	0.286942	0.0170386	-0.193197	1.651E-13	2.240E-14	1.053E-13
14	**β234**	-0.194162	0.160049	1.76603	1.726E-13	2.341E-14	1.100E-13
15	**β112**	-0.377868	0.261411	6.02484	2.146E-12	2.911E-13	1.368E-12
16	**β113**	1.43831	-0.936653	2.1818	1.775E-13	2.407E-14	1.131E-13
17	**β114**	1.34995	-0.67677	0.233209	7.543E-13	1.023E-13	4.809E-13
18	**β223**	0.73663	-0.915391	-1.24951	1.441E-12	1.955E-13	9.188E-13

19	**β224**	1.22807	-1.46276	-4.86298	1.645E-12	2.231E-13	1.048E-12
20	**β334**	0.0890372	-0.123587	-1.00754	1.980E-13	2.685E-14	1.262E-13

Table 7: Statistical Validation of t-Statistic and P-Value

		t-Statistic			**P-Value**		
S/No.	**βi**	**Coefficient of Consolidation**	**Coefficient of Compressibility**	**Compressive Strength**	**Coefficient of Consolidation**	**Coefficient of Compressibility**	**Compressive Strength**
1	**β1**	8.5398E+14	9.7184E+14	7.7941E+14	1.054E-230	9.484E-223	4.548E-230
2	**β2**	8.5297E+14	3.6199E+14	6.3765E+14	1.074E-230	9.702E-225	1.129E-228
3	**β3**	7.9789E+14	6.1493E+14	3.8593E+14	3.126E-230	2.017E-228	3.484E-225
4	**β4**	7.5755E+14	6.4028E+14	3.0422E+14	7.169E-230	1.057E-228	1.567E-223
5	**β12**	-2.2534E+11	-1.5892E+03	-2.7696E+12	1.909E-173	6.165E-187	7.037E-191
6	**β13**	-5.6387E+11	1.7985E+13	-7.6501E+12	8.075E-180	7.041E-204	6.129E-198
7	**β14**	-7.8816E+11	1.2794E+13	-1.8597E+12	3.804E-182	1.637E-201	4.121E-188
8	**β23**	-6.7063E+11	6.1194E+12	2.3070E+12	5.039E-181	2.181E-196	1.310E-189
9	**β24**	-1.0000E+12	1.0466E+13	1.7184E+12	8.431E-184	4.072E-200	1.458E-187
10	**β34**	-6.6593E+11	8.0239E+12	1.2814E+12	5.639E-181	2.857E-198	1.597E-185
11	**β123**	-4.8350E+11	-7.1857E+03	1.0285E+13	9.455E-179	1.989E-181	5.382E-200
12	**β124**	6.9657E+12	-2.4159E+03	-6.7321E+12	2.746E-197	7.699E-206	4.739E-197
13	**β134**	1.7379E+12	7.6080E+11	-1.8354E+12	1.218E-187	6.695E-182	5.087E-188
14	**β234**	-1.1250E+12	6.8366E+12	1.6050E+13	1.281E-184	3.704E-197	4.350E-203
15	**β112**	-1.7610E+11	8.9815E+11	4.4041E+12	9.859E-172	4.704E-183	4.211E-194
16	**β113**	8.1044E+12	-3.9414E+03	1.9283E+13	2.435E-198	3.057E-209	2.309E-204
17	**β114**	1.7898E+12	-6.6942E+03	4.8497E+11	7.609E-188	6.277E-197	9.009E-179
18	**β223**	5.1116E+11	-4.7391E+03	-1.3600E+12	3.883E-179	1.577E-194	6.159E-186
19	**β224**	7.4675E+11	-6.5573E+12	-4.6381E+12	9.021E-182	7.219E-197	1.839E-194
20	**β334**	4.4977E+11	-4.6577E+03	-7.9830E+12	3.008E-178	2.081E-194	3.100E-198

4. Conclusion

This study successfully optimized the strength and durability properties of Abuja laterite stabilized with Ordinary Portland Cement and Rice Husk Ash for structural earth block production.

Mineralogical analysis confirmed the high pozzolanic potential of RHA due to its silica content. The natural laterite exhibited low plasticity and favorable characteristics for stabilization. The incorporation of OPC and RHA significantly improved consolidation behaviour by reducing compressibility and enhancing stiffness.

Compressive strength results demonstrated that moderate RHA replacement combined with adequate OPC content produced stabilized blocks with strengths suitable for structural applications. Computational optimization using Scheffé's fourth-degree polynomial model yielded highly reliable predictive equations, with R^2 values exceeding 97%.

Overall, the study confirms that properly proportioned OPC–RHA blends can enhance both mechanical performance and sustainability of lateritic earth blocks.

5. Recommendations

Based on the findings of this study, the following recommendations are proposed:

1. Moderate RHA replacement (approximately 15%) combined with about 30% OPC is recommended for structural earth block production, balancing strength and consolidation performance.
2. Future research should investigate long-term durability under environmental exposure conditions such as wet–dry and freeze–thaw cycles.
3. Field-scale production and pilot construction projects should be undertaken to validate laboratory findings under real service conditions.
4. Further microstructural analysis using SEM or advanced imaging techniques is recommended to better understand pore structure evolution in stabilized laterite.
5. Standardization bodies should consider incorporating agricultural waste-based stabilization approaches into sustainable construction guidelines in Nigeria.

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