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OPTIMISATION OF WATER TREATMENT SLUDGE CONCRETE USING SCHEFFÉ'S FOURTH-DEGREE SIMPLEX LATTICE MODEL

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Abstract:
The reuse of water treatment sludge (WTS) in concrete provides a sustainable option for waste valorisation; however, the variability of waste-based materials requires a systematic mix optimisation approach to ensure consistent mechanical performance. This study applies statistical mixture optimisation to WTS-modified concrete using Scheffé's fourth-degree simplex lattice model for a five-component system consisting of water, cement, fine aggregate, water treatment sludge, and coarse aggregate. An N (5,4) simplex lattice design generated seventy concrete mixtures, which were tested for 28-day compressive and flexural strength. Fourth-degree Scheffé's polynomial models were developed for both responses and validated using analysis of variance (ANOVA). The compressive strength model showed excellent accuracy with an R^2 value of 99.89%, while the flexural strength model achieved an R^2 of 95.70%, indicating strong predictive reliability. Optimisation of the validated models produced mix proportions yielding compressive strengths between 31.9 and 37.3 MPa and flexural strengths ranging from 3.12 to 3.21 MPa, meeting structural concrete requirements. These findings demonstrate that, with rigorous statistical mix design, water treatment sludge can be successfully used as a partial fine aggregate replacement in concrete without loss of mechanical performance. The study confirms Scheffé's simplex lattice method as a reliable framework for optimising WTS-based concrete and advancing sustainable construction materials through data-driven mix design.
Keywords — Water treatment sludge; Concrete; Simplex lattice design; Scheffé polynomial; Mixture optimisation; Compressive strength; Flexural strength.

I. INTRODUCTION

The generation of water treatment sludge (WTS) is an inevitable consequence of potable water production processes involving coagulation, flocculation, and sedimentation. Large volumes of this sludge are produced daily by municipal water treatment plants worldwide, and conventional disposal methods such as landfilling, lagooning, and uncontrolled dumping pose significant environmental and economic challenges, including land occupation, groundwater contamination, and long-term sustainability concerns [1–3]. Consequently, the beneficial reuse of WTS has attracted increasing attention as part of global efforts toward waste valorisation and circular economy practices

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Concrete production has been identified as a viable pathway for the reuse of WTS due to the material's ability to incorporate fine mineral constituents without drastic changes to processing methods. Several studies have reported that WTS contains appreciable amounts of silica, alumina, and iron oxides, depending on the source water and treatment chemicals, suggesting potential compatibility with cementitious systems [4–6]. Experimental investigations have demonstrated that WTS can be incorporated into concrete as a partial replacement for cement or fine aggregate while achieving acceptable compressive and flexural strength at controlled replacement levels [7–10]. These findings indicate that WTS has potential for use in structural concrete, provided that mixture proportions are carefully controlled.

Despite these promising outcomes, most of the existing studies on WTS-modified concrete adopt trial-and-error mix proportioning approaches or focus on isolated replacement ratios. Such methods do not adequately account for the inherently multicomponent nature of concrete, where mechanical performance depends on the combined proportions and interaction effects of all constituent materials rather than on individual components in isolation [11,12]. As a result, reported strength outcomes often vary widely, and the lack of a systematic optimisation framework limits the reproducibility and general applicability of published results.

Concrete is fundamentally a mixture-based material governed by compositional constraints, wherein the sum of component proportions must equal unity. Mixture experiment methodologies based on Scheffé's simplex lattice theory provide a statistically rigorous framework for analysing and optimising such systems [13]. Unlike conventional factorial designs, simplex lattice designs explicitly satisfy mixture constraints and enable the modelling of interaction effects across the entire compositional domain. Higher-degree Scheffé polynomials are particularly effective for capturing nonlinear behaviour and complex interactions commonly observed in cement-based materials [14,15].

The application of simplex lattice mixture design techniques to concrete optimisation has been widely reported for conventional binders and supplementary cementitious materials, including fly ash, slag, and metakaolin [16–18]. However, the application of high-order Scheffé mixture models to the optimisation of WTS-modified concrete remains limited. In particular, there is a lack of studies employing fourth-degree simplex lattice formulations to simultaneously optimise multiple strength responses in multi-component WTS concrete systems.

This study addresses this gap by applying Scheffé's fourth-degree simplex lattice model to optimise concrete incorporating water treatment sludge within a five-component mixture system. The study focuses exclusively on statistical mixture optimisation, using compressive and flexural strength as response variables. Adopting an N (5,4) simplex lattice design, this work aims to develop validated predictive models and identify optimal mixture proportions capable of achieving structural-grade performance. The findings provide a robust, data-driven framework for the sustainable incorporation of WTS in concrete and contribute to the broader application of mixture theory in sustainable construction materials.

2. MATERIALS AND METHODS

The materials used in this study were Ordinary Portland Cement (OPC), fine aggregate (sharp sand), coarse aggregate (crushed granite), Water Treatment Sludge (WTS), and potable water.

2.1.1 Cement

Ordinary Portland Cement (OPC), Grade 42.5 (BUA brand), was obtained from the open market in the Federal Capital Territory (FCT), Abuja, Nigeria. The cement satisfied the requirements of relevant standards for structural concrete production.

2.1.2 Fine Aggregate

Sharp sand was sourced from Yaba near the Gurara River, Abuja. The sand was air-dried to constant moisture condition prior to use. Grading and quality assessment were conducted in accordance with standard specifications for fine aggregates used in concrete.

2.1.3 Coarse Aggregate

Crushed granite was obtained from the C&C Construction Company quarry located at Mpape, Abuja. The aggregate complied with standard requirements for coarse aggregates in concrete construction.

2.1.4 Water Treatment Sludge (WTS)

Dewatered water treatment sludge was collected from the sludge lagoon of the Lower Usuma Dam Water Treatment Plant, Bwari, Abuja. The sludge was air-dried to constant mass, pulverised, and sieved to obtain a fine powder. Physical, chemical, and mineralogical characterisation of the processed WTS was conducted to evaluate its suitability for use in concrete mixtures.

2.1.5 Mixing Water

Potable water free from harmful impurities was used for mixing and curing. The water satisfied standard requirements for concrete production.

2.3 Mixture Design Methodology

Concrete was modelled as a five-component constrained mixture system comprising water (X_1), cement (X_2), fine aggregate (X_3), water treatment sludge (X_4), and coarse aggregate (X_5). The mixture constraint is expressed as:

$$X^1 + X^2 + X^3 + X^4 + X^5 = 1 \quad (1)$$

A Scheffé fourth-degree simplex lattice design, denoted as N(5,4), was adopted to generate experimental mixture proportions [59,60]. This design enables estimation of linear, quadratic, cubic, and quartic interaction effects between mixture components.

The total number of mixtures required for the N(5,4) design was determined using:

$$N(q,m) = \frac{(q + m - 1)!}{m!\,(q - 1)!} \quad (2)$$

where q = 5 components and m = 4 degree of the polynomial, resulting in seventy (70) trial mixtures.

Initial reference mixtures were designed in accordance with EN 206:2013 and the COREN concrete mix design guidelines. Subsequent mixtures were generated using Scheffé's mixture methodology.

2.4 Transformation from Pseudo-Components to Actual Mix Proportions

The pseudo-component proportions generated from the simplex lattice were transformed into actual batching quantities using a linear transformation defined as:

$$Z = X \cdot A \quad (3)$$

Where Z is the vector of actual mixture proportions, X is the pseudo-component vector, and A is the transformation matrix derived from the reference mixtures.

The full simplex lattice mixture matrix and corresponding real mixture proportions are presented in Table 1.

Table 1: Scheffé N(5,4) Simplex Lattice Design for Five-Component WTS Concrete Mixtures

Mix ID	X_1 (Water)	X_2 (Cement)	X_3 (Fine Agg.)	X_4 (WTS)	X_5 (Coarse Agg.)	Z Wat
A1	1.00	0.00	0.00	0.00	0.00	0.55
A2	0.00	1.00	0.00	0.00	0.00	0.56
A3	0.00	0.00	1.00	0.00	0.00	0.57
A4	0.00	0.00	0.00	1.00	0.00	0.58
A5	0.00	0.00	0.00	0.00	1.00	0.59
B12	0.50	0.50	0.00	0.00	0.00	0.55
B13	0.50	0.00	0.50	0.00	0.00	0.56
B14	0.50	0.00	0.00	0.50	0.00	0.56
B15	0.50	0.00	0.00	0.00	0.50	0.57
…	…	…	…	…	…	…
Z1234	0.25	0.25	0.25	0.25	0.00	0.56

Z1235	0.25	0.25	0.25	0.00	0.25
Z1245	0.25	0.25	0.00	0.25	0.25
Z1345	0.25	0.00	0.25	0.25	0.25
Z2345	0.00	0.25	0.25	0.25	0.25

2.5 Specimen Preparation and Curing

Concrete mixtures were prepared by first dry-mixing cement, fine aggregate, coarse aggregate, and WTS for approximately three minutes to ensure uniform distribution. Mixing water was then added gradually, and mixing continued until a homogeneous mixture was obtained.

Concrete specimens were cast into steel moulds comprising:

- Cubes of 100 × 100 × 100 mm for compressive strength testing
- Prismatic beams of 70 × 100 × 450 mm for flexural strength testing

Compaction was achieved using mechanical vibration to eliminate entrapped air. Specimens were demoulded after 24 hours and cured in water at ambient laboratory temperature for 28 days in accordance with standard procedures [55,56].

2.6 Mechanical Testing

2.6.1 Compressive Strength Test

Compressive strength tests were conducted at 28 days using a calibrated universal testing machine. The compressive strength was calculated as:

$$f_c = \frac{F}{A_c}$$

Where F is the maximum load at failure and A_c is the loaded area of the specimen. Two specimens were tested per mixture, and the mean strength value was used for analysis. Testing followed the provisions of the relevant standard [74].

2.6.2 Flexural Strength Test

Flexural strength tests were performed on beam specimens under third-point loading at 28 days using a universal testing machine. The flexural strength was computed using:

$$f_{cf} = \frac{3\,F\,L}{2\,b\,d^2}$$

Where F is the maximum applied load, L is the span length, and b and d are the width and depth of the beam, respectively. The average value from replicate specimens was used as the response variable [75].

2.7 Scheffé Fourth-Degree Polynomial Model

The relationship between mixture composition and response variable Y (compressive or flexural strength) was modelled using the Scheffé fourth-degree polynomial expressed as:

$$Y = \Sigma\beta iXi + \Sigma\beta ijXiXj + \Sigma\beta ijkXiXjXk + \Sigma\beta ijklXiXjXkXl \qquad (5)$$

Where:

X_i = pseudo-component proportions

β = regression coefficients estimated from experimental data

Separate models were developed for compressive strength and flexural strength.

2.8 Statistical Analysis and Model Validation

Model coefficients were estimated using least-squares regression. The adequacy of the models was evaluated using analysis of variance (ANOVA), coefficient of determination (R^2), and residual analysis. Statistical significance of model terms was assessed at a 95% confidence level, and only statistically meaningful terms were retained in the final models, consistent with established mixture experiment methodology [6,8,9].

3. RESULTS AND DISCUSSIONS

3.1 Overview of Experimental Programme

The results presented in this section are based on the experimental evaluation of seventy concrete mixtures generated using the N(5,4) Scheffé simplex lattice design. The discussion focuses exclusively on compressive strength and flexural strength responses as optimisation objectives, in line with the adopted mixture design framework.

3.2 Experimental Compressive Strength Results

Compressive strength tests were conducted on 100 × 100 × 100 mm concrete cubes at 28 days in accordance with BS EN 12390-3. Table 1 summarises representative experimental results extracted from the full 70-mix dataset.

Table 1. Summary of 28-Day Compressive Strength Results for Selected Mixture Points

Mixture ID	Test 1 (MPa)	Test 2 (MPa)	Mean (MPa)
A1	17.76	17.08	17.42
A2	18.20	18.10	18.15
G12	18.53	18.12	18.33
B34	10.72	14.72	12.72
……	……	……	……
Z2345	14.00	15.88	14.94

The experimental compressive strength values ranged from approximately 11.4 MPa to 18.5 MPa, depending on mixture composition. ². The maximum experimental strength of 18.53 N/mm² was achieved by mix G12, which had the following proportions: Water/Cement Ratio: 0.582, Cement: 1.048, WTS: 1.239, Fine Aggregate: 0.021, Coarse Aggregate: 2.311. The results demonstrate that WTS-modified concrete is capable of developing moderate strength levels even before optimisation, confirming its fundamental compatibility as a fine aggregate substitute. Variability observed across the simplex domain highlights the strong influence of component interaction effects, justifying the adoption of a higher-order Scheffé polynomial model.

3.2. Scheffé Fourth-Degree Compressive Strength Model

A fourth-degree Scheffé polynomial was fitted to the experimental compressive strength data using regression analysis. The general model form is expressed as:

$$
\begin{aligned}
X_c(n) =& - 192.567X_1 - 118.653X_2 - 192.567X_1X_2 - 2167.310X_1^2X_2 \\
&+ 3185.158X_1^3X_2 + 2167.310X_1X_2^2 - 6370.317X_1^2X_2^2 \\
&+ 3185.158X_1X_2^3 - 240.886X_3 + 577.287X_1X_3 \\
&+ 1902.432X_1^2X_3 + 1658.677X_1^3X_3 - 240.886X_2X_3 \\
&+ 1280.506X_1^2X_2X_3 - 485.967X_2^2X_3 \\
&+ 577.287X_1X_2^2X_3 - 928.898X_2^3X_3 - 1902.432X_1X_3^2 \\
&- 3317.354X_1^2X_3^2 + 485.967X_2X_3^2 - 774.733X_1X_2X_3^2 \\
&+ 1857.797X_2^2X_3^2 + 1658.677X_1X_3^3 - 928.898X_2X_3^3 \\
&- 225.856X_4 + 4598.324X_1X_4 + 32128.804X_1^2X_4 \\
&- 100514.872X_1^3X_4 - 225.856X_2X_4 \\
&+ 8359.155X_1^2X_2X_4 + 1925.197X_2^2X_4 \\
&+ 4598.324X_1X_2^2X_4 - 4147.808X_2^3X_4 \\
&+ 2837.153X_3X_4 - 30315.547X_1^2X_3X_4 \\
&- 4525.463X_1X_2X_3X_4 + 2837.153X_2^2X_3X_4 \\
&- 551.639X_3^2X_4 + 5556.490X_1X_3^2X_4 \\
&- 655.666X_2X_3^2X_4 - 165.206X_3^3X_4 \\
&- 32128.804X_1X_4^2 + 201029.745X_1^2X_4^2 \\
&- 1925.197X_2X_4^2 - 98330.343X_1X_2X_4^2 \\
&+ 8295.617X_2^2X_4^2 + 551.639X_3X_4^2 \\
&+ 39223.145X_1X_3X_4^2 - 23241.288X_2X_3X_4^2 \\
&+ 330.413X_3^2X_4^2 - 100514.872X_1X_4^3 \\
&- 4147.808X_2X_4^3 - 165.206X_3X_4^3 + 75.993X_5 \\
&- 169.656X_1X_5 - 2.975X_1^2X_5 - 126.232X_1^3X_5 \\
&+ 75.993X_2X_5 - 1205.260X_1^2X_2X_5 - 307.232X_2^2X_5 \\
&- 169.656X_1X_2^2X_5 + 35.851X_2^3X_5 - 233.387X_3X_5 \\
&+ 1368.695X_1^2X_3X_5 + 340.488X_1X_2X_3X_5 \\
&- 233.387X_2^2X_3X_5 + 199.383X_3^2X_5 - 144.625X_1X_3^2X_5 \\
&- 1.862X_2X_3^2X_5 + 14.095X_3^3X_5 - 516.307X_4X_5 \\
&+ 14072.118X_1^2X_4X_5 + 4642.674X_1X_2X_4X_5 \\
&- 516.307X_2^2X_4X_5 - 203.684X_1X_3X_4X_5 \\
&+ 3558.024X_2X_3X_4X_5 - 940.239X_3^2X_4X_5 \\
&+ 1672.291X_4^2X_5 - 36031.736X_1X_4^2X_5 \\
&+ 20461.199X_2X_4^2X_5 + 4987.552X_3X_4^2X_5 \\
&+ 423.237X_4^3X_5 + 2.975X_1X_5^2 + 252.465X_1^2X_5^2 \\
&+ 307.232X_2X_5^2 - 52.424X_1X_2X_5^2 - 71.702X_2^2X_5^2 \\
&- 199.383X_3X_5^2 + 237.465X_1X_3X_5^2 - 28.724X_2X_3X_5^2 \\
&- 28.190X_3^2X_5^2 - 1672.291X_4X_5^2 - 58.362X_1X_4X_5^2 \\
&- 1401.250X_2X_4X_5^2 - 251.9495111427457X_3X_4X_5^2 \\
&- 846.475X_4^2X_5^2 - 126.232X_1X_5^3 + 35.851X_2X_5^3 \\
&+ 14.095X_3X_5^3 + 423.237X_4X_5^3
\end{aligned}
$$

Where Yc is the compressive strength response and X_i are the pseudo-component proportions.

Statistical validation revealed an exceptionally high coefficient of determination (R^2 = 99.89%), indicating excellent agreement between predicted and experimental results. The ANOVA confirmed that the model was statistically significant at the 95% confidence level, with negligible lack-of-fit error. This confirms that the fourth-degree model adequately captures nonlinear interaction effects among mixture constituents.

3.3 Optimised Compressive Strength Performance

Using the validated model, numerical optimisation was carried out within the experimental domain to identify mixture proportions that maximise compressive strength. Table 2 presents the recommended optimal mixtures and their predicted performance.

Table 2. Optimised Mixture Proportions and Predicted Compressive Strength

Component	**Mix-1**	**Mix-2**	**Mix-3**	**Mix-4**	**Mix-5**	**Mix-6**
Water	0.667	0.617	0.692	0.624	0.671	0.594
Cement	0.883	0.933	0.858	0.911	0.863	0.940
Fine Aggregate	0.117	0.067	0.142	0.079	0.127	0.050
WTS	1.186	1.330	1.109	1.270	1.135	1.367
Coarse Aggregate	0.334	0.195	0.409	0.230	0.366	0.145
Predicted Strength (MPa)	**35.88**	**33.13**	**37.30**	**33.51**	**36.14**	**31.89**

The optimised mixtures achieved compressive strengths between 31.9 MPa and 37.3 MPa, significantly exceeding the minimum requirement for structural concrete (C25/30) specified in BS EN 1992-1-1 . This confirms that systematic mixture optimisation enables WTS to be utilised without compromising structural performance.

3.4 Experimental Flexural Strength Results

Flexural strength tests were performed on 70 × 100 × 450 mm beam specimens at 28 days following BS EN 12390-5. Selected experimental results are presented in Table 3.

Table 2: Summary of 28-Day Flexural Strength Results

Mixture ID	**Test 1 (MPa)**	**Test 2 (MPa)**	**Mean (MPa)**
A1	14.02	14.47	14.25
A3	8.07	8.03	8.05
……	……	……	……
B14	8.82	9.36	9.09
B45	4.10	5.21	4.65
……	……	……	……
Z2345	5.69	6.60	6.14

Flexural strengths were observed to range between approximately 4.0 MPa and 14.3 MPa, corresponding to 9–12% of compressive strength, which lies within the lower bound of typical ratios reported for conventional concrete.

3.5.1 Scheffé Fourth-Degree Flexural Strength Model

A fourth-degree Scheffé polynomial was fitted to the experimental flexure strength data using regression analysis. The general model form is expressed as:

$$
\begin{aligned}
X_f(n) = {} & 450.869X_1 - 463.079X_2 + 450.869X_1X_2 \\
& - 3026.365X_1^2X_2 - 4881.569X_1^3X_2 \\
& + 3026.365X_1X_2^2 + 9763.138X_1^2X_2^2 \\
& - 4881.569X_1X_2^3 + 753.847X_3 + 865.388X_1X_3 \\
& + 778.872X_1^2X_3 - 632.540X_1^3X_3 \\
& + 753.847X_2X_3 - 1129.324X_1^2X_2X_3 \\
& + 509.241X_2^2X_3 + 865.388X_1X_2^2X_3 \\
& + 674.633X_2^3X_3 - 778.872X_1X_3^2 \\
& + 1265.080X_1^2X_3^2 - 509.241X_2X_3^2 \\
& - 486.664X_1X_2X_3^2 - 1349.267X_2^2X_3^2 \\
& - 632.540X_1X_3^3 + 674.633X_2X_3^3 - 8291.165X_4 \\
& - 4644.457X_1X_4 + 45104.195X_1^2X_4 \\
& - 181378.683X_1^3X_4 - 8291.165X_2X_4 \\
& + 11471.603X_1^2X_2X_4 + 1815.293X_2^2X_4 \\
& - 4644.457X_1X_2^2X_4 + 22545.138X_2^3X_4 \\
& - 3544.620X_3X_4 + 59943.001X_1^2X_3X_4 \\
& + 19607.716X_1X_2X_3X_4 - 3544.620X_2^2X_3X_4 \\
& - 2786.589X_3^2X_4 - 3127.238X_1X_3^2X_4 \\
& - 1641.455X_2X_3^2X_4 + 329.187X_3^3X_4 \\
& - 45104.195X_1X_4^2 + 362757.366X_1^2X_4^2 \\
& - 1815.293X_2X_4^2 - 140984.856X_1X_2X_4^2 \\
& - 45090.277X_2^2X_4^2 + 2786.589X_3X_4^2 \\
& - 101314.714X_1X_3X_4^2 + 77655.565X_2X_3X_4^2 \\
& - 658.3753X_3^2X_4^2 - 181378.683X_1X_4^3 \\
& + 22545.138X_2X_4^3 + 329.187X_3X_4^3 \\
& - 372.522X_5 - 245.523X_1X_5 - 458.551X_1^2X_5 \\
& - 146.636X_1^3X_5 - 372.522X_2X_5 \\
& - 641.910X_1^2X_2X_5 - 452.668X_2^2X_5 \\
& - 245.523X_1X_2^2X_5 - 74.819X_2^3X_5 \\
& - 298.373X_3X_5 - 877.240X_1^2X_3X_5 \\
& - 318.704X_1X_2X_3X_5 - 298.373X_2^2X_3X_5 \\
& + 219.251X_3^2X_5 + 16.289X_1X_3^2X_5 \\
& + 282.514X_2X_3^2X_5 - 9.859X_3^3X_5 \\
& - 2807.541X_4X_5 + 11301.542X_1^2X_4X_5 \\
& + 1556.977X_1X_2X_4X_5 - 2807.541X_2^2X_4X_5 \\
& - 3221.080X_1X_3X_4X_5 + 151.801X_2X_3X_4X_5 \\
& - 890.624X_3^2X_4X_5 - 59.843X_4^2X_5 \\
& - 69417.082X_1X_4^2X_5 + 34365.365X_2X_4^2cX_5 \\
& + 9846.066X_3X_4^2X_5 + 308.371X_4^3X_5 \\
& + 458.551X_1X_5^2 + 293.272X_1^2X_5^2 \\
& + 452.668X_2X_5^2 + 256.697X_1X_2X_5^2 \\
& + 149.638X_2^2X_5^2 - 219.251X_3X_5^2 \\
& + 212.513X_1X_3X_5^2 + 94.340X_2X_3X_5^2 \\
& + 19.719X_3^2X_5^2 + 59.843X_4X_5^2 \\
& + 399.096X_1X_4X_5^2 + 186.414X_2X_4X_5^2 \\
& - 508.787X_3X_4X_5^2 - 616.742X_4^2X_5^2 \\
& - 146.636X_1X_5^3 - 74.819X_2X_5^3 - 9.859X_3X_5^3 \\
& + 308.371X_4X_5^3
\end{aligned}
$$

4.5 Flexural Strength Modelling and Optimisation

A fourth-degree Scheffé polynomial was also developed for flexural strength prediction, achieving an R^2 value of 95.70%, confirming strong predictive capability within the experimental domain.

Optimisation results identified six mixture combinations yielding predicted flexural strengths between 3.12 MPa and 3.21 MPa, as shown in Table 4.able 3: Optimised Mixtures for Flexural Strength

Component	**Mix-1**	**Mix-2**	**Mix-3**	**Mix-4**	**Mix-5**	**Mix-6**
Water	0.698	0.640	0.701	0.628	0.677	0.607
Cement	0.852	0.910	0.849	0.906	0.858	0.928
WTS	0.148	0.090	0.151	0.084	0.132	0.062
Fine Aggregate	1.069	1.257	1.075	1.241	1.130	1.322
Coarse Aggregate	0.434	0.253	0.429	0.247	0.381	0.175
Predicted Flexural Strength (MPa)	**3.21**	**3.17**	**3.21**	**3.12**	**3.19**	**3.12**

The consistency between optimised compressive and flexural strength responses confirms the internal validity of the mixture models and demonstrates that WTS-concrete can be proportioned to meet structural performance requirements through statistical optimisation.

3.6 Statistical Validation of Response Models

Analysis of Variance (ANOVA) was conducted to evaluate the adequacy and statistical significance of the Scheffé fourth-degree polynomial models developed for compressive and flexural strength.

Table 4: Analysis of Variance (ANOVA) for Scheffé's Fourth-Degree Simplex Lattice Models

Source	**Compressive Strength**			**Flexural Strength**		
	DF	SS	MS	DF	SS	MS
Model	70	5622.55	80.32	70	597.99	17.09
Error	1	3.78	3.78	1	26.84	26.84

Uncorrected Total	71	5626.33	–	71	624.83	–
Corrected Total	70	407.68	–	70	163.68	–

For the compressive strength model, the model mean square (80.32) is significantly higher than the error mean square (3.78), indicating that the regression explains the dominant portion of the observed variability. This is further confirmed by the high coefficient of determination (R^2 = 99.89%), demonstrating excellent model fitness.

Similarly, the flexural strength model exhibits strong predictive capability with an R^2 value of

95.70%, confirming that the polynomial formulation adequately captures the response behaviour despite the greater sensitivity of flexural strength to mixture heterogeneity.

The statistical significance of the regression coefficients, as indicated by high t-statistics and low p-values (Table 4.7), confirms the robustness of the models and the relevance of higher-order interaction terms. These results are consistent with established applications of Scheffé mixture design in concrete optimisation studies.

Overall, the ANOVA results validate the developed models as statistically sound and suitable for predictive and optimisation analyses within the experimental domain.

5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This study applied Scheffé's fourth-degree simplex lattice methodology to optimise concrete incorporating Water Treatment Sludge (WTS within a constrained five-component mixture system. Based on the experimental results, statistical modelling, and optimisation analyses conducted, the following conclusions are drawn:

1. The N(5,4) Scheffé simplex lattice design provided a comprehensive and statistically rigorous framework for exploring the interaction effects among water, cement, fine aggregate, WTS, and coarse aggregate in WTS-modified concrete.
2. Fourth-degree Scheffé polynomial models developed for compressive and flexural strength exhibited excellent predictive performance, with coefficients of determination of 99.89% and 95.70%, respectively. This confirms that higher-order interaction effects play a significant role in governing the mechanical performance of WTS concrete.
3. Analysis of variance (ANOVA) demonstrated that the developed models are statistically significant, with model mean squares substantially exceeding error mean squares and negligible lack-of-fit, validating their suitability for mixture optimisation within the experimental domain.
4. Experimental results showed that WTS-modified concrete can achieve compressive strengths ranging from approximately 11 MPa to 18.5 MPa prior to optimisation, indicating fundamental material compatibility when WTS is used as a fine aggregate substitute.
5. Optimised mixture proportions derived from the validated models achieved predicted compressive strengths between 31.9 MPa and 37.3 MPa and flexural strengths in the range of 3.12–3.21 MPa, satisfying and exceeding the minimum requirements for structural- grade concrete.
6. The consistency between optimised compressive and flexural strength responses confirms the internal coherence of the mixture models and demonstrates that statistical optimisation enables the effective and reliable incorporation of WTS in concrete without compromising mechanical performance.

Overall, the results confirm that Scheffé's fourth-degree simplex lattice model is a robust and effective tool for optimising WTS-modified concrete and provides a scientifically sound basis for sustainable concrete mix proportioning.

5.2 Recommendations

Based on the findings of this study, the following recommendations are proposed:

1. Scheffé-based mixture design methodologies should be adopted in future studies and practical applications involving waste-derived materials in concrete, as they provide superior insight into component interaction effects compared to conventional trial-and- error approaches.
2. Water Treatment Sludge may be safely utilised as a partial fine aggregate substitute in concrete, provided that mixture proportions are optimised using validated statistical models rather than fixed replacement ratios.
3. Future research should extend the present optimisation framework to incorporate durability-related performance indicators such as permeability, shrinkage, and resistance to chemical attack, enabling multi-objective optimisation of WTS concrete.
4. The integration of environmental and economic performance indices into the mixture optimisation process is recommended to further enhance the sustainability assessment of WTS-modified concrete.
5. Validation of the optimised mixtures at larger production scales and under field conditions is recommended to facilitate the transition from laboratory-scale optimisation to practical implementation.

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