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Autonomous Human Body Control, Part XII: Serum Calcium Concentration Control during the Dialysis Process using PD-PI and 2DOF-5 Controllers Compared with a PI Controller

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Abstract:
This paper investigates the tuning of PD-PI and 2DOF-5 controllers from the second generation of PID controllers to control the serum calcium concentration of the human body during dialysis. The proposed controllers are tuned using a hybrid approach based on zero/pole cancellation and the application of the MATLAB optimization toolbox to minimize an ITAE performance index. The serum calcium concentration process is modeled using pre-published dialysis data and the tuning results of the proposed controllers are presented and applied to generate the step time response for reference input tracking of a specific level within the normal limits of the serum calcium concentration. The characteristics of the step time responses are compared with those of a conventional PI controller. The best controller for the control of the human serum calcium concentration during dialysis is assigned.

Keywords **— Autonomous human body control, serum calcium concentration control during dialysis, PI controller, PD-PI controller, 2DOF-5 controller. controller tuning.**

I. INTRODUCTION

This is the twelfth paper of a series of research papers oriented towards the study of autonomous human body control to help reducing the human suffering due to the deficiencies in the operating organs. This paper deals with the control of the serum calcium concentration during the dialysis operation where it helping in the automatic treatment of hypocalcemia to lie within the normal level of 8.6 to 10.3 mg/dL [1]. Decreased calcium concentration levels (< 8.6 mg/dL) results in some symptoms on the human health such as: dry scaly skin, brittle nails, coarse hair, muscle crumps, confusion, memory loss, delirium, depression and hallucinations [2]. From here emerges the importance of regulating the serum calcium concentration to be within the normal. Unfortunately there a severe shortage in the efforts of automatic control of human serum calcium. Here are some of the research efforts in presenting the serum calcium measurement and kinetics since 2001:

Wong et al. (2001) determined the total calcium and magnesium concentrations with colorimetric and point assay procedures. They concluded that strong correlations were found between calcium, magnesium and zinc in seminal plasma [3]. Kariazis et al. (2002) outlined that low dialysate calcium concentration is used to prevent or treat hemodialysis (HD) induced hypercalcemia. They performed hemodialysis studies with 1.25, 1.75 and 1.5 mmol/L over pre-assigned periods. They concluded that the blood pressure dropped during the last 2 hours in both low and medium dialysate calcium groups [4]. Qotch et al. (2007) developed a kinetic model of calcium mass balance during dialysis for a single-compartment with variable volume. They claimed that their model predicted the serial plasma calcium contractions very well. They presented plasma calcium profiles calculated using the data of other researchers for time up to 250 min [5]. Pakkyara et al. (2010) outlined that hypercalcemia may follow hypocalcemia in the course of acute renal failure (acute kidney injury). They presented a case of young male who developed acute renal failure due to strenuous exercise inducing rhabomyolysis [6].

Tomas et al. (2014) investigated the association between albumin –adjusted serum calcium concentration and type 2 diabetes in subjects with high cardiovascular risk. They concluded that an increase in serum calcium concentration is associated with an increased risk of of type 2 diabetes in individuals with cardiovascular risk [7]. Inagoma et al. (2017). Outlined that in patients on maintenance dialysis, increased calcium levels are associated with poor prognosis. They concluded that serum adjusted calcium levels at dialysis

initiation were demonstrated to be associated with all-cause mortality after dialysis initiation [8]. Sokoh et al. (2019) outlined that conversion of dialysate calcium concentration from 2.5 to 2.75 mEq/L increased intradialytic calcium loading and serum total and ionized calcium levels. They concluded that conversion of dialysate calcium concentration from 2.5 to 2.75 mEq/L resulted in expected changes in calcium concentration loading based on predialysis calcium concentration and the dialysate calcium concentration should be personalized based on clinical factors [9]. Lint, Pirklbauer et al. (2020) outlined that calcium loading is associated with cardiovascular risk in hemodialysis patients. They provided clinical evidence for rapidly accessible and exchangeable calcium pol involved in interdialytic calcium regulation and for the osteocalcin as a potential biomarker [10].

Gao et al (2022) outlined that preoperative serum calcium, preoperative alkaline phosphatase and preoperative intact parathyroid hormone were significant indicators of hypocalcemia in patients with secondary hyperparathyroidism after parathyroidectomy [11]. Yamada et al. (2024) outlined that conversion of dialysate calcium from 3.0 to 2.6 mEq/L lowered serum calcium levels leading to decreased calcium loading among patients undergoing hemodialysis. They presented serum calcium profile in mg/dL for 7 months period [12]. Du et al (2025) evaluated clinical features of decreased serum calcium compared with two non-hypocalcemia in systematic lupus erthematosus (SLE) patients. They concluded that SLE patients with decreased calcium had strong disease activity and required longer treatment time for remission [13].

II. THE CONTROLLED CALCIUM CONCENTRATION DURING DIALYSIS AS A PROCESS

Building a dynamic model for purpose of automatic control purposes requires relatively large number of data points to achieve high accuracy of the model. Unfortunately, this was not the case in the present work since points available in serum calcium kinetics had very limited number of data points. One of the available data was in the work of Sokoh et al. [9], they presented graphical profile of the variation of the serum calcium concentration over a time period of 180 min corresponding to dialysate calcium change from 2.5 to 2.75 mg/dL. I have used their clinical data at 24 week after dialysate calcium conversion. I referred the data to the original value at zero time to assign the changes in the serum calcium concentration against the change of the input dialysate calcium dose (2.75-2.5=0.25 mg/dL). The transfer function, $G_p(s)$ = Ca(s) / DCa(s)] was identified by the author using an ITAE performance index [14] and the MATLAB optimization toolbox [15]. With a correlation coefficient of 0.9936, the serum calcium concentration is identified as a delayed first-order process having $G_p(s)$ given by:

$$G_p(s) = K \exp(-T_d s) / (Ts + 1) \qquad (1)$$

Where:

K = process gain = 0.9152 (dimensionless)

T_d = process delay time = 22.459 min

T = process time constant = 30.543 min

The delay time of this process is about 73.5 % of its time constant meaning considerable time delay which is expected to affect the performance of its controller (especially PID) in terms of its stability and tuning [16]. This factor puts a challenge on proposing some controllers from the second generation of PID controllers presented by the author since 2014. The time response profile for an input dose change of 0.25 mg/dL dialysate calcium is shown in Fig.1 as generated by the step command of MATLAB [17] for the serum calcium change.

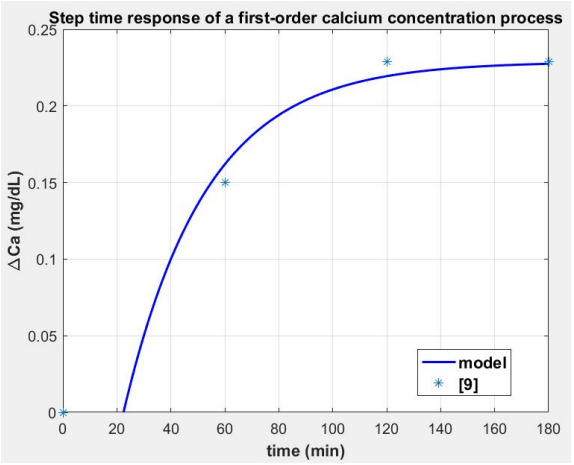

Fig.1 Step time response of the calcium concentration as a process during dialysis.

COMMENTS:

- The serum calcium concentration during dialysis is a stable process.
- Maximum overshoot: zero
- Settling time: 140 min

III. SERUM CALCIUM CONTROL DURING DIALYSIS USING A PI CONTROLLER

- As a reference for control system characteristic comparison, a conventional PI controller from the first-generation of PID controllers is proposed to control the serum calcium concentration.
- To simplify dealing with time delay in the controlled process, the exponential term in Eq.1 is replaced with a rational 3/ 4 Pade approximation in the form [18]:

$$\exp(T_d s) = N(s)/D(s) \quad (2)$$

where:

$$N(s) = -T_d{}^3 s^3 + 60 T_d{}^2 s^2 - 360 T_d s + 840$$

$$D(s) = T_d{}^4 s^4 + 16 T_d{}^3 s^3 + 120 T_d{}^2 s^2 + 480 T_d s + 840$$

- Combining Eqs.1 and 2 gives the serum calcium concentration process transfer function as:

$$G_p(s) = N(s)/[(Ts+1)D(s)] \quad (3)$$

- The PI controller transfer function has the transfer function, $G_{PI}(s)$ given by:

$$G_{PI}(s) = K_{pc1} + (K_{i1}/s) = (K_{pc1} s + K_{i1})/s \quad (4)$$

Where K_{pc1} and K_{i1} are the proportional and integral gains of the PI controller.

- The two elements: $G_{PI}(s)$ and $G_p(s)$ in a single-loop control system are cascaded in series.
- Multiplying $G_{PI}(s)$ by $G_p(s)$ gives the open-loop transfer function of the control system.
- Now, the closed-loop transfer function of the control system, $G_{PI}(s)G_p(s)/[1+G_{PI}(s)G_p(s)]$ can be derived using Eqs.3 and 4.
- The controller parameters K_{pc1} and K_{i1} in the closed-loop transfer function are tuned through the minimization of an ITAE performance index [14] using the MATLAB optimization toolbox [15]. The result of this process gives the tuned PI controller parameters as:

$K_{pc1} = 1.043573$, $K_{i1} = 0.0251608$ (5)

- The step time response of a desired calcium concentration change of 3 mg/dL (corresponding to 9 mg/dL calcium concentration absolute value) when using a PI controller is shown in Fig.2. Lower and upper limits of 8.6 and 10.3 mg/dL respectively [1] are shown also in Fig.2.

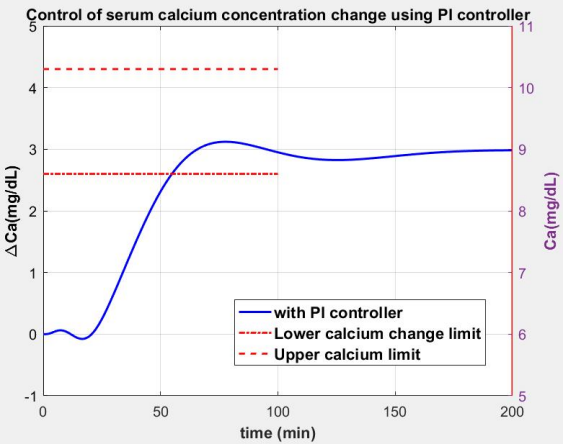

Fig.2 Calcium concentration control during dialysis using a PI controller.

COMMENTS:

- Maximum overshoot: 4.014 %
- Maximum undershoot: -0.0774 mg/dL
- Settling time with ± 2 % tolerance: 165.1 min
- Settling time within the calcium concentration limits: 54.852 min
- Steady-state error: zero

IV. SERUM CALCIUM CONCENTRATION CONTROL USING A PD-PI CONTROLLER

- The PD-PI controller was introduced by the author in April 2014 to control first-order delayed processes as one of the second generation of PID controllers introduced by the author since 2014 [19]. The PD-PI controller is consisted of two cascaded control modes: PD and PI set in the single-loop block diagram of a linear control

system just after the error detector. It has a transfer function $G_{PDPI}(s)$ given by:

$$G_{PDPI}(s) = G_{PD}(s) G_{PI}(s) \quad (6)$$

Where:

$$G_{PD}(s) = K_{pc2} + K_{d2} s = K_{pc2}[1 + (K_{d2} / K_{pc2}) s]$$

$$G_{PI}(s) = K_{pc3} + (K_{i3} / s)$$

Where K_{pc2}, K_{d2}, K_{pc3} and K_{i3} are the gain parameters of the PD-PI controller.

- Multiplying $G_{PDPI}(s)$ of Eq.6 by $G_p(s)$ of Eq.3 and applying the zero/pole cancellation technique [20] gives the following relationship between the PD control element parameters:

$$K_{d2} = T K_{pc2} \quad (7)$$

- The closed-loop transfer function of the control system incorporating the PD-PI controller is derived as $M_4(s) = G_{PDPI}(s)G_p(s)/[1+ G_{PDPI}(s)G_p(s)]$. This transfer function will be function of only three gain parameters for the PD-PI controller: K_{pc2}, K_{pc3} and K_{i3}.
- The three gain parameters are tuned using the ITAE performance index [14] and the MATLAB optimization toolbox [15] and Eq.7 is used to provide the fourth gain Kd2 of the controller. The tuned four parameters of the PD-PI controller are given by:

$K_{pc2} = 0.249981$, $K_{d2} = 7.635174$

$K_{pc3} = 0.507087$, $K_{i3} = 0.107845$ (8)

- The step time response of a desired calcium concentration change of 3 mg/dL (corresponding to 9 mg/dL calcium concentration absolute value) when using a PD-PI controller is shown in Fig.3.

COMMENTS:

- Maximum overshoot: zero (compared with 4.014 % for the PI controller).
- Maximum undershoot: -0.105 (compared with -0.0774 mg/dL for the PI controller).
- Settling time with ± 2 % tolerance: 78.64 min (compared with 165.1 min for the PI controller).
- Settling time within the calcium concentration limits: 59.157 min (compared with 54.852 min for the PI controller).
- Steady-state error: zero

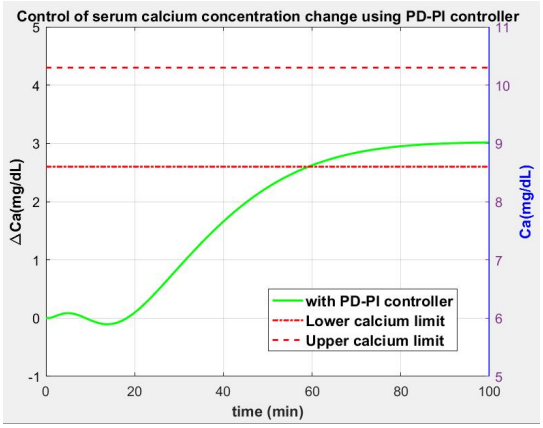

Fig.3 Calcium concentration control during dialysis using a PD-PI controller.

V. SERUM CALCIUM CONCENTRATION CONTROL USING A 2DOF-5 CONTROLLER

- The 2DOF controller was introduced by the author in August 2015 to control a highly oscillating second-order-like process as one of the second generation of PID controllers introduced by the author since 2014 [21]. The author different structures of the 2DOF controller to control various dynamic processes. Fig.4 shows one of those structures [22] with control modes having transfer functions $G_{c1}(s)$ in the forward path of the block diagram of the control system and $G_{c2}(s)$ in the feedback path. 5 in 2DOF-5 refers to the type of $G_{c1}(s)$ and $G_{c2}(s)$ of the 2DOF controller as follows:

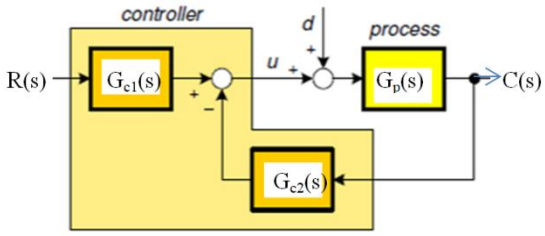

Fig.4 Structure of a 2PDF controller [22].

$$G_{c1}(s) = K_{pc4} + K_{d4} s$$

$$G_{c2}(s) = K_{pc5} + (K_{i5} / s) \quad (9)$$

Where K_{pc4}, K_{d4} are the gains of the PD control mode of the 2DOF-5 controller and K_{pc5} and K_{i5} are the gains of its PI control mode.

- The four gain parameters of the 2DOF-5 controllers are tuned by minimizing an ITAE performance index [14] using the MATLAB optimization toolbox [15] providing the following tuned controller parameters:

$$K_{pc4} = 1.150940 \ , \ K_{d4} = 0.921996$$
$$K_{pc5} = 0.004960 \ , \ K_{i5} = 0.000840 \qquad (10)$$

- The transfer function derived from the block diagram in Fig.4 and the tuned controller gain in Eq.10 produce the step time response of the serum calcium concentration shown in Fig.5.

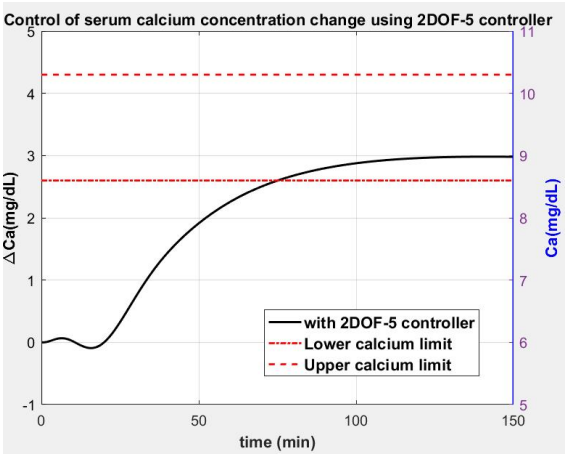

Fig.5 Calcium concentration control during dialysis using a 2DOF-5 controller.

COMMENTS:

- Maximum overshoot: zero (compared with 4.014 % for the PI controller).
- Maximum undershoot: -0.105 (compared with -0.0774 mg/dL for the PI controller).
- Settling time with ± 2 % tolerance: 78.64 min (compared with 113 min for the PI controller).
- Settling time within the calcium concentration limits: 75 min (compared with 54.852 min for the PI controller).
- Steady-state error: zero

VI. COMPARISON OF TIME BASED CHARACTERISTICS

Graphical Comparison:

- The time-based characteristics of the control systems incorporating the controllers proposed to control the serum calcium concentration during dialysis are compared graphically through the step time response as depicted in Fig.6 for a desired calcium concentration change of 3 mg/dL (9 mg/dL absolute value).

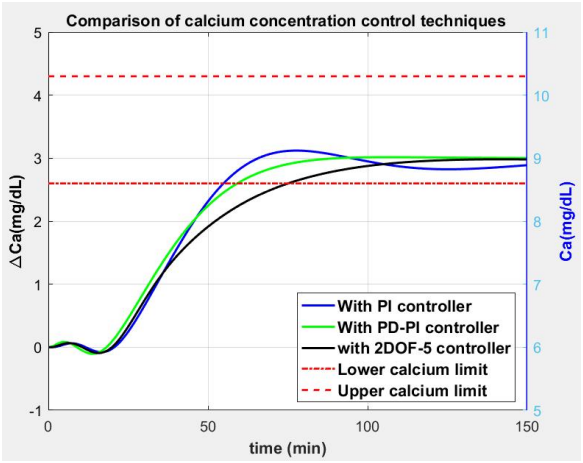

Fig.6 Graphical characteristic comparison of serum calcium control during dialysis.

Numerical Comparison:

- Numerical comparison for the time-based characteristics of the step time response for reference input tracking of the control system with the proposed controllers is presented in Table 1 with comparison with the application of a conventional PI controller used to control the same calcite concentration process.

TABLE 1
TIME-BASED CHARACTERISTICS FOR CONTROLLERS COMPARISON

Controller	PI	PD-PI	2DOF-5
OS_{max} (%)	4.014	0	0
US_{max} (mg/dL)	-0.077	-0.105	-0.105
$T_{s2\%}$ (min)	165.1	78.64	113
$T_{slimits}$ (min)	54.85	59.157	75
e_{ss} (mg/dL)	0	0	0

OS_{max}: Maximum overshoot.
US_{max}: Maximum undershoot.
$T_{s2\%}$: Settling time for 2 % tolerance.
$T_{slimits}$: Settling time for calcium concentration limits.

VII. CONCLUSIONS

- The research work presented in this research paper handled the tuning of two controllers from the second generation of PID controllers proposed to control the serum calcium concentration of a human being during dialysis compared with a PI controller from the first generation of PID controllers.
- The controlled serum calcium concentration as a process was identified as a delayed first-order one. Its parameters were identified using few data from a previous research work. .
- The proposed controllers were tuned using hybrid approach based on applying the zero/pole cancellation and the MATLAB optimization toolbox.
- All the proposed controllers succeeded to eliminate completely the steady-state error of the control system.
- The proposed PD-PI and 2DOF-5 controllers succeeded to eliminate completely the maximum percentage overshoot of the control system compared with 4.014 % for the PI controller.
- All the proposed controllers succeeded to reduce the settling time of the control system (with respect to the 2 % tolerance) to values in the range: $78.65 \leq T_{s2\%} \leq 113$ min compared with 165.1 min for the PI controller.
- The PD-PI controller succeeded to reduce the settling time of the control system to 78.64 min compared with 165.64 min for the PI controller.
- The best controller is the PD-PI controller based on the least maximum overshoot and settling time.
- Future work is required to produce more accurate dynamic model for the serum calcium concentration process through collecting more data during dialysis following the application of the step input of the dialysate calcium.

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DEDICATION

PROF. GAMAL ABDULAZIZ AHMED

I have the honor to dedicate this work to Prof. Gamal Abdulaziz Ahmed. Why?

- He is the author of more than 60 books on all sciences of the Arabic Language.
- He wrote 12 research papers in scientific journals and conferences.
- He wrote about 2200 articles in Egyptian, Gulf and European newspapers.
- Prof. Gamal is a truly lover of Egypt and of any researcher working seriously to raise its name in the international domain.
- In the last month's he didn't miss any of my published papers with intensive comments and encouraging words.
- Thanks Professor for your love to Egypt and for the time you have given to support my published papers.

BIOGRAPHY

Galal Ali Hassaan

- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his B.Sc. and M.Sc. from Cairo University in 1970 and 1974.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
- Now with the Faculty of Engineering, Cairo University, EGYPT.
- Research on Automatic Control, Mechanical Vibrations, Mechanism Synthesis and History of Mechanical Engineering.
- Published more than 350 research papers in international journals and conferences.
- Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.
- Chief Justice of the International Journal of Computer Techniques.
- Member of the Editorial Board of some international journals including IJET.
- Reviewer in some international journals.
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