# Formulation of Voltage Stability Second Order Governing Equation and Arithmetic Moving Average for Transmission Line Prediction and Improvement.

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**ABSTRACT –** The Nigeria 330kV integrated power system currently consists of the existing network, national independent power projects (NIPP), and independent power producers (IPP). This network consists of generating stations, transmission lines, and buses. Consequently, the Nigerian power system is gradually transforming into a complex interconnected network of different components. This complexity is because of the deregulation of the electricity industry and the expansion of the network by the National Independent Power Project (NIPP) and Independent Power Producers (IPP) to meet the increasing energy demand. A balance between active and reactive power will ensure a reliable electric power system for the consumer at the receiving end. For low power factor of the system essentially indicates inefficient delivery of active power to the load due to reactive power losses. This paper study considered the application of predictive optimizers with the view to assess various voltage stability indices (VSI), particularly fast voltage stability index (FVSI), line stability index (LMN), line stability factor (LQP), voltage stability index (LD) and novel line stability index (NLSI), are presented to predict the proximity of the line close to voltage collapse. Following the predictive pattern, three (3) predictive indices (NLSI, LMN, FVSI) captured the voltage collapse behaviour in their respective predictive order, while voltage stability (LD) and line stability factor (LQP) are far from capturing predictive behaviour for voltage collapse, because of its slow dynamic response capacity to the system at abnormal conditions or violations. The mean absolute percentages error (MAPE) indicates NLSI as better and faster in terms of performance capacity, followed by line stability index (LMN) and fast voltage stability index (FVSI) as the slow predictive index for voltage stability under study. While five (5) yearly moving average techniques captured 11 numbers of voltage collapse for the year 2021-2024,10 numbers of voltage collapse for the year 2025-2029. The indices, NLSI, LMN and FVSI show predictive behaviour for system voltage collapse. The Novel line stability index (NLSI) has better and faster predictive characteristics capacity for determining voltage instability especially, Shiroro (generator-bus), Okpai (generator–bus) Kumbotso (load bus), Jos (load-bus), Markudi (load-bus) Damaturu (load-bus), Ikeja-west (load-bus), Ikot-Ekpene (load bus. The research paper also introduced the application of artificial neural network (ANN), to measure system parameters performance, correlation, and validation with input data (FVSI, LMN, LQP, LD, NLSI). The obtained quantitative of R = 0.9993 while the validity value was 0.9993 which agrees with the data input for parameters relationship. [1,2]

***Keywords- Voltage Collapse, Instability, Predictive Optimizer, Formulation, Arithmetic, Transmission lines, Power Grid, Second Order Governing Equation, Voltage Stability***

## I. INTRODUCTION

Generally power system analysis is known to generate at low voltages and step up for further transmission, that is from 11KV or 15KV to 330KV. The step up voltage also need to be stepped down for distribution requirement for purpose of consumer utility. [3]

One type of system instability that results from heavily loaded system is voltage collapse. Voltage collapse is manifested in the form of slow variation in the system operating point because of continuous increase in load which eventually leads to a corresponding decrease in magnitude of the voltage. This continuous decrease eventually results in a sharp acceleration of the process until there is zero voltage in the system [4, 5].

Voltage collapse is a situation that leads to low drops in voltage and eventually power system blackout, these phenomena have been identified as primary power system faults that must be avoided at all costs. This is due to the magnitude of its negative impact on power system infrastructure and in turn, it is highly detrimental in terms of economic impact to the society [6]

## II. PROBLEM FORMULATION

The problem of transmission voltage collapse prediction requires determination of voltage parameters and stability indices which include:

- Fast voltage Stability Index (FVSI)
- Line Stability Index (LMN)
- Line Stability Factor (LQP)
- Voltage Stability Index (LD)
- Novel Line Stability Index (NLSI)

**Formulation of Second Order Governing Equation for Voltage Stability Predictions**

This paper shows the mathematical technique to calculate line voltage stability, particularly a line connected between node x and y that shows long transmission line from its outages condition or collapse point. The procedure provides a simple pattern of assessing voltage stability analysis it has the capacity to provide reduced calculation time needed to prevent voltage collapse.

The following governing equations parameters for the study case are presented as;

$V_x$: Sending-end voltage
$V_y$: Receiving-end voltage
$\delta_x$: Sending-end voltage angle.
$\delta_y$: Receiving-end voltage angle.
$P_x$: Sending-end real power at bus x
$P_y$: Receiving-end real power at bus y
$Q_x$: Sending-end reactive power at bus x
$Q_y$: Receiving-end reactive power at bus y
$Y_{xy} = (G+JB)$: line admittance between bus x and y
$\theta$: Line admittances angle
$R+jx$: Line impedance between bus x and y
Consider bus -x taken as a reference bus under this analysis and investigation.

$B_{xy}$: Susceptance of the transmission line (between bus-x and bus-y)

Thus, the line current ($I_L$) is calculated and given as:

$$I_{Line} = \left(V_x - V_y\right)Y_{xy} = VY \qquad (1)$$

Similarly,

$$I_{line} = \left(\frac{S_x}{V_y}\right) = \frac{P_x - jQ_x}{V_x\angle -\delta_x} \qquad (2)$$

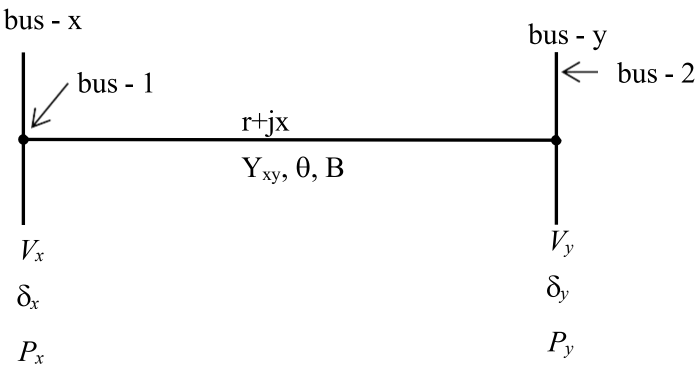


Figure 1: Formulation of a simple two-bus system representation.

$$p_x - jQ_y = I_{line}.V_x\angle -\delta_x$$

Let bus-x be taken as the reference bus, while the line current ($I_{Line}$) is calculated as follows given by;

$$I_{Line} = \left(V_x - V_y\right)Y_{xy} \qquad (3)$$

The line currents, ($I_{line}$) can be determined using the receiving – apparent power at bus - y

$$I_{line} = \left(\frac{S_y}{V_y}\right) = \frac{P_y - jQ_y}{V_y\angle -\delta_y} \qquad (4)$$

Rearranging and formulating the given equation (3) and (4) to introduce impedance angle (θ), to account for the nodes (x and y admittance of two buses is given as;

$$\left(V_x - V_y\right)Y_{xy} = \frac{}{\angle -} \qquad (5)$$

or

$$P_y - jQ_y = \left(V_x - V_y\right)Y_{xy}.V_y\angle -\delta_y \qquad (6)$$

or

$$P_y - jQ_y = V_x Y_{xy} V_y\angle -\delta_y - V_y Y_{xy}\angle -\delta_y \qquad (7)$$

or

$$P_y - jQ_y = \left|V_x V_y Y_{xy}\right|\angle\left(\theta - \delta y\right) - \left|V_y\right|^2\left|V_{xy}\right|\angle\theta \qquad (8)$$

Separating real and imaginary part of equation (7) Real part:

$$P_y = \left|V_x V_y Y_{xy}\right|\cos\left(\theta - \delta y\right) - \left|V_y\right|\left|V_{xy}\right|\cos\theta \qquad (9)$$

and

imaginary part:

$$Q_y = \left|V_x V_y Y_{xy}\right|\sin\left(\theta - \delta_y\right) + \left|V_y\right|^2\left|V_{bus}\right|\sin\theta \qquad (10)$$

Substitute (8) into (7):

$$P_y = \left|V_x V_y Y_{xy}\right|\cos\left(\theta - \delta y\right) + \left|V_y\right|\left|Y_{xy}\right|\cos\theta \qquad (11)$$

And

$$p_y - jQ_y = \left|V_x V_y Y_{xy}\right|\angle\left(\theta - \delta_y\right) - \left|V_y\right|^2\left|Y_{bus}\right|\angle\theta \qquad (12)$$

That is given as,

$$\left|V_x V_y Y_{xy}\right|\cos\angle\left(\theta - \delta y\right) - \left|V_y\right|^2.\left|Y_{bus}\right|\cos\theta - \quad i_{Q_y} =$$

$$\left|V_x V_y Y_{xy}\right|\angle\left(\theta - \delta y\right) - \left|V_y\right|^2.\left|Y_{bus}\right|\angle\theta \qquad (13)$$

To establish the relationship between $V_R$ and $Q_y$ gives the followings as;

From 12:

$$Qy = -\left|V_x V_y Y_{xy}\right|\sin\left(\theta - \delta y\right) + \left|V_y\right|^2.\left|Y_{xy}\right|\sin\theta$$

Transmission Line representation of a simple two (2) – bus network

$Y_{xy}\sin\theta$ $\quad Y_{xy}\sin\theta$ $\quad Y_{xy}\sin\theta$

Let, $Y_{xy} = Y_y \qquad (15)$

$$\frac{Q_y}{Y_y\sin\theta} = -\left|V_x V_y\right|\frac{\sin\left(\theta - \delta_y\right)}{\sin\theta} + \left|V_y\right|^2 \qquad (16)$$

or

$$\left|V_y\right|^2 - \left|V_x.V_y\right|\frac{\sin\left(\theta - \delta_y\right)}{\sin\theta} - \frac{Q_y}{\left|Y_y\right|\sin\theta} = 0 \qquad (17)$$

Since, $\delta_y$ is very small it is assumed to be zero ( 0 ) this means that the whole term of $\sin\left(\theta - \delta y\right)/\sin\theta$ evidently can be eliminated to give $Q$ as: (18)

$$\left|V^y\right| - \left|V^x.V^y\right|.1 - \frac{Y_y\sin\theta}{\left|_y\right|} = 0$$

Let, $B_{xy} = Y_y \sin\theta$ then the new equation can be rewritten as;

$$\left|V_y\right|^2 - \left|V_x V_y\right| - \frac{Q_y}{B_{xy}} = 0 \qquad (19)$$

Equation (19) can therefore be represented as 2[nd] order quadratic polynomial given as:

$ax^2 + bx + c = 0$ (general expression for quadratic equation) (20)

That is,

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (21)$$

Taking quadratic expression of $V_y$ from the established equation of roots $V_y$ are presented as;

$$V_y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (22)$$

Where, the coefficients values are obtained as;

$\left\{a = 1,\ b = V_x,\ c = -\frac{Q_y}{B_{xy}}\right\}$, substitute into the general quadratic equations; (23)

That is,

$$V_{y1,2} = \frac{-\left|V_x\right| \pm \sqrt{\left|Vx\right|^2 - 4\times 1\times\left(\frac{-Q_y}{B_{xy}}\right)}}{2\times 1} \qquad (24)$$

Or

$$V_{y1,2} = \frac{-V_x}{2} + \sqrt{\frac{\left|Vx\right|^2}{2} + 4.\frac{Q_y}{B_{xy}}} \qquad (25)$$

Simplifying the quadratic equations to obtain as;

$$V_{y1,2} = \frac{-V_x}{2} + \sqrt{\frac{\left|Vx\right|^2}{2} + 4.\frac{Q_y}{B_{xy}}} \qquad (26)$$

$$V_{y1,2} = \frac{-V_x}{2} \pm \sqrt{\left|\frac{Vx}{2}\right|^2 + 4.\frac{Q_y}{B_{xy}}} \quad (27)$$

Similarly, $V_{y1,2}$ can be rewritten as $\left(when\ \alpha = \frac{V_x}{2}\right)$;

$$V_{y1,2} = -\alpha \pm \sqrt{|\alpha|^2 + 4\times.\frac{Q_y}{B_{xy}}} \quad (28)$$

or

$$V_{y1,2} = -\alpha \pm \sqrt{|\alpha|^2 + 4\times.\frac{Q_y}{B_{xy}}} \quad (29)$$

Each root characteristics equation is given as;

$$\left.\begin{aligned} V_{y1} &= -\alpha - \sqrt{|\alpha|^2 + 4\times.\frac{Q_y}{B_{xy}}} \\ &\text{and} \\ V_{y2} &= -\alpha + \sqrt{|\alpha|^2 + 4\times.\frac{Q_y}{B_{xy}}} \end{aligned}\right\} \quad (30)$$

Line data parameter verifications are conducted to ascertain the root parameters of the established equation for the two-bus system under study in the 330kV transmission line.

The line data for the two 2 buses network are:

$Y_{xy}$ = 1.244 (line admittance between bus x and y)

$Q_y = 1.5$ (Reactive power at the receiving-end)

$V_x$ = 0.87 (voltage value at the sending-end)

θ = 0.88 (line impedance angle)

$B_{xy}$ = 1.244 sin θ (susceptance between bus x and bus y at line impedance)

or

$B_{xy}$ = 1.244 ×0.7707

or

$B_{xy}$ = 0.8887

$$V_{y1} = \frac{0.87}{2} + \sqrt{\left(\frac{0.87}{2}\right)^2 + 4\times\frac{1.5}{0.8887}}$$

$$V_{y1} = \frac{0.87}{2} + \sqrt{\left(\left(\frac{0.87}{2}\right)^2 + 4\times\frac{1.5}{0.8887}\right)}$$

or

$$V_{y1} = 0.435 + \sqrt{\left((0.435)^2 + 46.75143\right)}$$

or

$$= -0.435 + \sqrt{(0.189225 + 6.75143)}$$

or

$$= -0.435 + \sqrt{6.940655}$$

$$= -0.435 + 2.63451 = 2.19951$$

$$V_{y1} = 2.1995$$

or

$$V_{y2} = -0.435 - 2.63451$$

or

$$V_{y2} = -3.06951$$

**Arithmetic Moving Average Technique**

Arithmetic moving average (also known as a simple moving average, SMA) commonly used as statistical calculation in time series analysis and forecasting). It is computed by taking the average of a series of data points within a specific period. The formula for calculation of SMA are defined, the parameter definition is stated as.

$$M = \frac{1+2+...+}{} \quad (31)$$

Where,

SMA: the simple moving average.

$X_1, X_2, ....X_n$ are the data points within the chosen period.

*n*: the number of data points in the period

**ARIMA (Auto Regressive Integrated Moving Average):**

ARIMA is a complex time series forecasting model that combines autoregressive (AR) and moving average (MA) components with differencing to make the data stationary. The ARIMA model is represented as: ARIMA (p, d, q) where:

P: the order of the autoregressive components.

d: the degree of differencing (the number of times differencing is applied to make the data stationary)

q the order of the moving average components.

The governing equations for ARIMA involves the following steps:

Differencing: make the time series data stationary by differencing it the times until it becomes stationary.

The difference series is denoted as $Y_t$.

Auto-regression (AR) components: Fit an autoregressive model of order p to the differences data $Y_t$. This involves estimating coefficients for lagging values of $Y_t$ and expressing $Y_t$ as a function of its past values.

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + .. + \phi_p Y_{t-p} + \varepsilon_t \quad (32)$$

Where; $\phi_1, \phi_2 \; ..... \; \phi_p$ are the autoregressive coefficient, and

$\varepsilon_t$, is the noise or error term.

**Moving Average (MA) Components**

In computing moving average model of order q to account for the lagged forecast errors ($\varepsilon_t$).

That is,

$$\varepsilon_t = \upsilon_1 \varepsilon_{t-1} + \upsilon_2 \varepsilon_{t-2} + .. + \upsilon_p \varepsilon_{t-p} \quad (33)$$

Where;

$\theta_1, \theta_2 \; ....., \; \theta_q$ are the moving average coefficient.

Forecasting: The ARIMA model is fitted using the forecasting future values of the time series.

ARIMA – model is widely used in time series analysis and forecasting, especially for data with trends and seasonally. The choice of p, d, and q values depends on the specific characteristics of the data and requires model selection technique. [7]

**Choice of Simple Moving Average for Forecasting**

Simple Moving Average (SMA) as a method for statistical prediction is simple and easy to understand and made use of just as the name implies, providing smoothing effect, reducing noise, and highlighting trends. It is less sensitive to outliers compared to other methods as well as its flexibility that allows the choosing of the window size for the analysis [8]

**Auto Regressive Integrated Moving Average (ARIMA)**

In the case of ARIMA which captures complex patterns and long-term trends in data, it requires stationarity of the data under investigation which may not necessitate differencing. Parameter estimation and model stationary can be challenging [9].

**Application Approach to Simple Moving Average, SM Technique**

Many types of data smoothing method are normally applied or in use but the commonest type is the simple moving average, SMA and i computed as;

$$\hat{y}_i = \frac{\sum_{j=i-k}^{i-k} y_j}{m} \quad (34)$$

Where;

$$K = \frac{m-1}{2}$$

$$K = \frac{-1}{2}$$

i: point location of the estimated moving average valu usually placed with respect to j:

j: point location of the observed time series data, and

m: length of the smoothing interval or number of points ove which the average is computed.

Alternatively, a simple moving average of order m is given by th sequence of arithmetic means as;

$$\frac{y_1 + y_2 + \ldots + y_m}{m};\ \frac{y_2 + y_3 + \ldots + y_{m+1}}{m};\ \frac{y_3 + y_4 + \ldots + y_{2+2}}{m} \quad (35)$$

The sum in the numerators of (34) are called totals of order m. whil (35 and 34) defines an interval centered around the point to b estimated. If m is an odd number, then the "estimated" $\hat{y}$ mus corresponds with the central point. If m is even, a set of value y wi be estimated that will be mid-way between adjacent observations However, the smoothing interval extends $(m-1)/2$ observation on either side of the estimated point, observations near the beginnin and the end of the sequence cannot be estimated if m is three (3), tha is the sequence (smoothed data) by two (one at both ends of data) ar noted strongly. [10]

Essentially, when data are given annually, monthly, or hourly, a moving average of order m is considered for analysis which is called an m-year moving average or m-hours moving average.

## III. CASE STUDY

**Overview of Voltage Collapse Cases in the 330kV Power Grid**

Cases of voltage collapse in the Nigeria 330kV power grid for the past 20 years that need a deeper study of power instability in the Nigeria power grid. A total of 315 cases of total voltage collapse were recorded between the years 2000 and 2019 which is 20 years. To achieve a stable grid the case of total voltage collapse must be eliminated. Table 1 is an overview of the total voltage collapse cases on the Nigeria 330kV power network over the past 20 years.

**Table 1: History of Voltage Collapse for the Period of 2000-2019**

| YEAR | PARTIAL COLLAPSE | TOTAL COLLAPSE | TOTAL CASES |
|---|---|---|---|
| 2000 | 6 | 5 | 11 |
| 2001 | 5 | 14 | 19 |
| 2002 | 32 | 9 | 41 |
| 2003 | 39 | 14 | 53 |
| 2004 | 30 | 22 | 52 |
| 2005 | 15 | 21 | 36 |
| 2006 | 10 | 20 | 30 |
| 2007 | 8 | 18 | 26 |
| 2008 | 16 | 26 | 42 |
| 2009 | 20 | 19 | 39 |
| 2010 | 20 | 22 | 24 |
| 2011 | 6 | 13 | 19 |
| 2012 | 8 | 16 | 24 |
| 2013 | 2 | 22 | 24 |
| 2014 | 4 | 9 | 13 |
| 2015 | 4 | 6 | 10 |
| 2016 | 6 | 22 | 28 |
| 2017 | 9 | 12 | 21 |
| 2018 | 1 | 12 | 13 |
| 2019 | 1 | 13 | 13 |

(Source: Power Holding Company of Nigeria (PHCN) Daily Operational Report).

**Table 2: Summary of History of Voltage Collapse for the Period of 2000 – 2019**

| YEAR | TOTAL CASES |
|---|---|
| 2000 | 5 |
| 2001 | 14 |
| 2002 | 9 |
| 2003 | 14 |
| 2004 | 22 |
| 2005 | 21 |
| 2006 | 20 |
| 2007 | 18 |
| 2008 | 26 |
| 2009 | 19 |
| 2010 | 22 |
| 2011 | 13 |
| 2012 | 16 |
| 2013 | 22 |
| 2014 | 9 |
| 2015 | 6 |
| 2016 | 22 |
| 2017 | 12 |
| 2018 | 12 |
| 2019 | 13 |

Source: (Source: Power Holding Company of Nigeria (PHCN) Daily Operational Report)

Analysis of the average Voltage Collapse can be obtained from the formula below:

$$= \frac{\sum =1}{\quad} \quad (36)$$

$$\% = \quad (37)$$

$$\% = \quad (38)$$

**Table 3: Average Voltage Collapse for the Period of 2000 – 2019**

| Year | Total Cases | Average Voltage Collapse |
|---|---|---|
| 2000 | 5 | 2% |
| 2001 | 14 | 4% |
| 2002 | 9 | 3% |
| 2003 | 14 | 4% |
| 2004 | 22 | 7% |
| 2005 | 21 | 7% |
| 2006 | 20 | 6% |
| 2007 | 18 | 6% |
| 2008 | 26 | 8% |
| 2009 | 19 | 6% |
| 2010 | 22 | 7% |
| 2011 | 13 | 4% |

| | | |
|---|---|---|
| 2012 | 16 | 5% |
| 2013 | 22 | 7% |
| 2014 | 9 | 3% |
| 2015 | 6 | 2% |
| 2016 | 22 | 7% |
| 2017 | 12 | 4% |
| 2018 | 12 | 4% |
| 2019 | 13 | 4% |
| Total | 315 | 100% |

Source: (TCN, 2020)

An acute deficiency in power generation will always manifest. This mismatch normally results in frequency fluctuation which could culminate in system instability.

An electric power grid is a system of electric power generation and transmission equipment/devices, including the lines – interconnected in a mesh (network) for efficient delivery of electricity.

Power systems are often interconnected to:

(i) Improve reliability and quality of power supply.

(ii) Reduce the spinning reserve requirement of individual systems.

(iii) Utilize the synergy offered by grid control and optimization devices.

Quite a few research works abound in the field of voltage stability which defines several Voltage Stability Margins (VSMs) in order to standardized power system networks. This study on VSMs is related to the concept of voltage collapse which is typically described by a voltage stability or collapse index at a point of possible line outage or blackout; thus, Voltage Collapse Index (VCI) typically describes the state of security of power system in the presence of possible severe contingency.

The network showing the modelled of 330Kv transmission system.

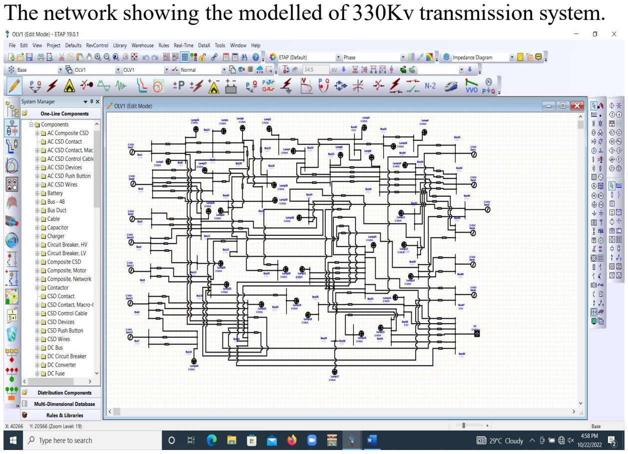

**Figure 2: Shows the Single-Line Representation of the Existing Steady Case (330KV Nigeria Network Not Simulated)**

## IV RESULT AND DISCUSSION

In order to determine the result of the study case, the two bus network was modeled to represent the transmission line for predicting voltage collapse.

**MATLAB Code for the Second Order quadratic polynomia Equation; as referred to figure 1.**

```
% Define parameters
Vx = -20:0.1:20; % Range of Vx values
Qy = 10; % Qy value
Bxy = 5; % Bxy value
% Calculate Vy1 and Vy2
Vy1 = -Vx/2 + sqrt((abs(Vx)/2).^2 + 4*Qy/Bxy);
Vy2 = -Vx/2 - sqrt((abs(Vx)/2).^2 + 4*Qy/Bxy);
% V_{(y1,2)} = -V_x/2 \pm \surd(|V_x|/2)^2 + 4*Q_y/B_xy
figure;
plot(Vx, Vy1, 'b', 'LineWidth', 2);
hold on;
plot(Vx, Vy2, 'r', 'LineWidth', 2);
grid on;
xlabel('Vx');
ylabel('Vy');
title('Plot of Vy1 and Vy2 as function of Vx');
legend('Vy1', 'Vy2', 'Location', 'best');
```

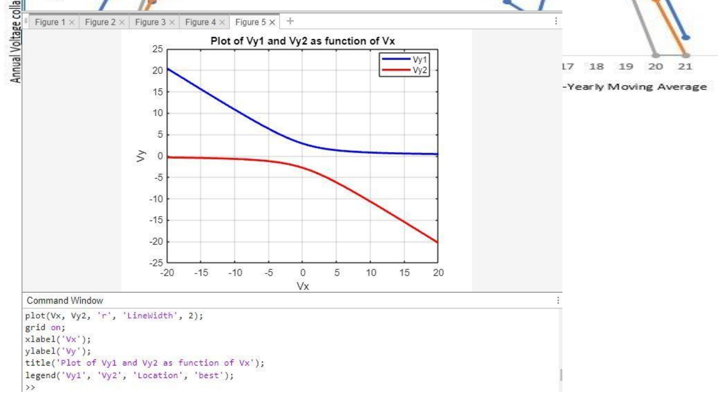


**Figure 2 shows plot of Vy1 and Vy2 as a Function of Vx**

Where is the sending end voltage, the receiving end voltage, receiving end reactive power and susceptance of the transmission line between bus x and bus y

The MATLAB code provided generates a plot showing the relationship between Vy1 and Vy2 as functions of Vx, based on the given parameters.

Vx, Qy, and Bxy represent the parameters of the equation $_{1,2} = -\frac{}{2} \pm \sqrt{\frac{||^2}{2} + 4 * \cdot}$.

- Vx is a range of values for the x-axis, covering from -20 to 20 with a step size of 0.1.

- Qy and Bxy are constants representing specific values for the parameters Qy and Bxy in the equation.

- The equation describes relationship between Vy1 and Vy2 with respect to Vx. It represents a mathematical model that can be used in engineering, particularly in the analysis of power systems, where voltage stability prediction is of critical concern.

The root of the equation defines the first voltage stability limit at $Vy_1$ to be 2.2, while the second voltage stability limit root given to be -3.1. This means that the first voltage stability roots predicted voltage stability at 2.2 while the second roots predicted slow voltage dynamic responses which is instability condition of the system condition. According to the statutory stability condition which defines voltage margin from zero (0) to one (1). Zero (0) defines lowest point of stability while one (1) define highest point of stability. This means that any predicted value beyond the maximum limits of 1 will be classified as unstable which may evidently lead to voltage collapse. The formulated 2nd order quadratic polynomial equations are characterized into two sets of roots to determine stability condition. For system planners and operators.

**Comparative Analysis of the Five (5) Predictive Indices (FVIS LMN, LQP, LD and NLSI)**

Voltage stability is a paramount consideration in the form of power system analysis, serving as a cornerstone for ensuring seamless and dependable operation of electrical grids. As the analysis unfolds, the primary focus lies in scrutinizing the intricacies of voltage stability through five pivotal predictive optimizers: FVSI, LMN, LQP, LD, and NLSI. These optimizers serve as invaluable tools, offering insights into the dynamic behavior of voltage levels within the grid infrastructure.

In essence, exploration into voltage stability through these predictive optimizers underscores the critical importance of proactive analysis and mitigation strategies in safeguarding the integrity and reliability of electrical grids.

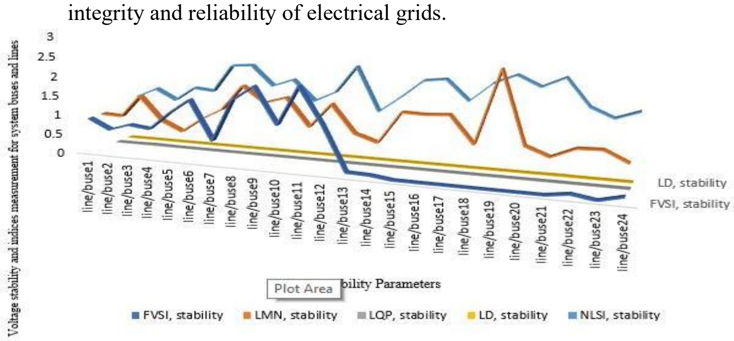


**Figure 3: Five (5) stability analysis indices for prediction of voltage collapse**

By scrutinizing data from 2000 to 2021, this analysis aims to track the expected number of voltage collapses, enabling a thorough examination of historical trends. Moreover, it serves as a pivotal resource for informing future planning.

"Figure 4 presents a graphical representation of a composite bar chart utilized in the annual voltage collapse analysis of 330kV networks.

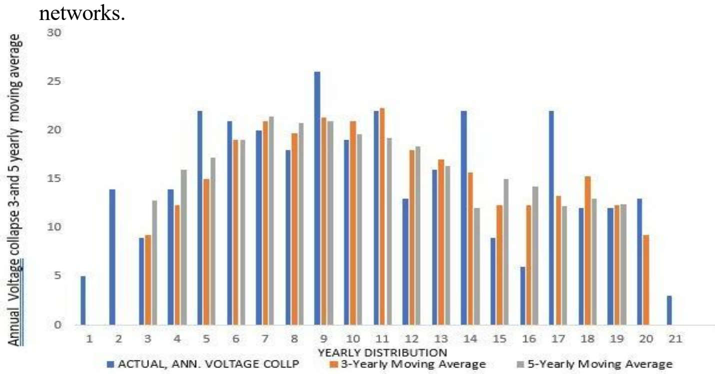


**Figure 4: Composite Bar Chart of Actual Annual Voltage Collapse, 3 and 5 yearly Moving Average Trend for Collapse Prediction on the 330kv Network.**

The chart employs both three-year and five-year arithmetic movin averages to assess the prediction of voltage collapses over the cours of each year. By examining the number of voltage collapses annually this analysis offers valuable insights into the stability and reliabilit of the network infrastructure.

**Figure 5: Composite line Graph of Actual Annual Voltage Collapse, 3 and 5 yearly Moving Average Trend for 330kv Network Assessment**

Figure 5 shows graphical representation of composite bar chart for annual voltage collapse analysis using three (3) and five (5) yearly arithmetic moving average for the assessment of 330kv networks prediction for number of voltages collapses annually. Twenty-one (21) historically data set were obtained as actual voltage collapse record which are examined as; (5, 14, 9, 14, 22, 21, 20, 18, 26, 19, 22, 13, 16, 22, 9, 6, 22, 9, 6, 22, 12, 12, 13 and 3) from the year (2000-2021). The arithmetic moving average technique was used to determine the number of voltage collapse while auto regressing historical prediction of voltage collapse for previous occurrence and into the future projection to secure system planning, system security for reliable power supply. Prediction of the expected number of voltage collapse using three (3) and five (5) yearly moving average are used to predicts the voltage collapse as; (12, 12, 12, 11, 11, 11, 11, 11, 10, 10, 10, and 10) and (11.1, 11.4, 11.2, 11, 10.8, 10.6, 10.4, 10.2 and 10) respectively from (2021-2032) projection. The results evidently shows that the highest number of expected numbers of voltage collapse was 12 using three (3) yearly moving average which evidently fall within the year 2021, 2022 and 2023 respectively. Then followed by subsequent year 2024, 2025, 2026, 2027, 2028 with 11 expected number of voltage collapses, and gradually becomes lowest in the year 2029-2032 with total expectation number of voltage collapse to be 10 while five years moving average techniques also captured 11 number of voltage collapse for the year 2021-2024, while 10 number of voltage collapse for the year 2025-2029.

## V. CONCLUSION

The Nigerian power network comprises a limited number of generating stations, predominantly situated in remote areas near raw fuel sources. These stations are often linked to load centers by extensive transmission lines. The generation, transmission, distribution, and marketing of electricity in Nigeria are statutory functions handled by the electricity utilities, notably the Power Holding Company of Nigeria, among others.

Currently, the installed generating capacity stands at approximately 12,522MW, with maximum dispatch capacity of about 4000MW, serving a population exceeding 200 million people. This represents a gross inadequacy in meeting the demand for electric power supply to consumers at receiving end. There is an urgent need to augment the current projected capacity of electricity supply to alleviate system overloads and prevent network collapses from occurring regularly.

Voltage stability is imperative for optimal system performance. Variations in load demand can trigger system overloads or disturbances that may lead to total outages or blackouts. Therefore, reliable power supply is crucial to enhance daily utility from the system buses.

In this regard, historical data on voltage collapse incidents has been gathered to assess network behavior. The study adopted the three-year and five-year moving average techniques to analyze the annual number of voltage collapses between 2000-2021 and 2021-2032. The predictive models indicate the highest number of expected voltage collapses to be 12, occurring in the years 2021, 2022, and 2023, followed by 11 collapses expected between 2024-2028, and 10 collapses predicted for 2029-2032.

This framework is termed as the "predictive optimizer," whivh has been characterized as a second-order quadratic polynomial. Its help to determine the receiving-end voltage (Vy) in relation to the sending-end voltage (Vx), as well as the reactive power at the receiving end (Qy). The study introduced the application of artificial neural network (ANN) to also measure system parameters and perform correlation, and validate input data (FVSI, LMN, LQP, LD, NLSI). Regression value (R) of 1 indicates an exact linear relationship, while R = 0 denotes no relationship between inputs and outputs (targets). The quantitative value of R = 0.9993 during validation demonstrates a high level of agreement with the dataset parameters' relationship.

Five voltage stability indices (NLSI, FVSI, LMN, LQP, LD) has been strategically employed to assess and predict the maximum capacity limit in each scenario of voltage collapse within the 330kV long transmission network. Among these indices, (NLSI, FVSI, and LMN) exhibit excellent predictive behaviors regardless of system voltage condition. In this study, the Novel Line Stability Index (NLSI) stands out superior and rapid predictive capabilities in identifying voltage instability. Notably, critical nodes such as Shiroro, Okpai, Kumbotoso, Jos, Makundi, Damaturu, Ikeja-west, Ikot-Ekpene, Ayede, Aja, and Egbin are classified as critically overloaded buses surpassing their maximum loadability limits are effectively identified by NLSI. while, LMN and FVSI serve as two other predictive indices contributing to the assessment. The study introduces the innovative use of simple moving average technique, which examines historical data, to predict and forecast voltage collapses. The incorporation of this technique enriches the predictive capabilities of the study, offering insights into potential system vulnerabilities and enhancing the overall reliability and resilience of the power network.

The research study has implemented application of three-year and five-year moving average techniques to ascertain expected number of voltage collapses from periods 2000-2021 and 2021-2032. This approach aim to facilitate reliable and efficient network planning, system security, and enhance operational efficiency for seamless power supply management, ultimately mitigating the risk of system voltage collapse.

## VI. APPENDIX A1.1

**Error Analysis For System Accuracy**

Using Mean Absolute Percent Error (MAPE)

From the comparison table in Appendix A1 we have

MAPE = $\frac{\Sigma(o-)}{}$ X 100

MAPE (FVSI/LMN) = $\frac{18.842475}{24}$ X 100 = 78.676%

MAPE (FVSI/LQP) = $\frac{14.99101231}{24}$ X 100 = 62.462%

MAPE (FVSI/LD) = $\frac{14.98115656}{24}$ X 100 = 62.421%

MAPE (FVSI/NSLI) = $\frac{23.8809768}{24}$ X 100 = 99.504%

NLSI > LMN > FVSI

From the above calculations, we rank the five optimizers as follows:

NLSI > LMN > FVSI >LQP > LD.

Which agrees with the graph in fig 4.5a

## APPENDIX A2

Solving for Reactive Power for the Nigerian 330kV Power Network

The Reactive Power needed in the Network is determined using the conventional governing reactive power equation ($Q_c$) as stated below

$Q_c = \frac{}{1}\sin(\cos^{-1} f_1) - \frac{}{2}\sin(\cos^{-1} f_2)$

Where P = 1300MW, =existing maximum load = Active power for the load

$Pf_1$ = 0.65 existing power factor, $Pf_2$ = 0.85 proposed power factor

$Q_c = \frac{1300}{0.65}\sin(\cos^{-1}(0.65)) - \frac{1300}{0.85}\sin(\cos^{-1}(0.85))$

= 2000 x Sin [0.863211] – 1529.41176470 x Sin [0.554811032]

= 2000 x 0.759933629 – 1529.41176470 x 0.52678268680

= 1519.867258 – 805.6676386321

=714.19961 ≈ 800MVAR

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