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RESTRAINED EDGE DOMINATION ON $S-$ VALUED GRAPHS

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ABSTRACT. In this paper, we introduce the notion of restrained edge domination on $S-$ valued graphs and study some properties.

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1. Introduction

In[6], the authors introduced the notion of $S-$ valued graphs, where S is a semiring. In graph theory, domination of graphs is the most powerful area of research for, it has several applications in other areas of sciences. It was inicticted by Berge [1]. In [7], the authors have studied the edge domination on $S-$ valued graphs. In this paper we discuss the notion of restrained edge domination on $S-$ valued graphs.

2. Preliminaries

In this section we recall some basic definitions that are needed for our work.

Definition 2.1. [4] *A semiring $(S,+,\cdot)$ is an algebraic system with a non-empty set S together with two binary operations $+$ and $\cdot$ such that*

(1) $(S,+,0)$ *is a monoid.*
(2) $(S,\cdot)$ *is a semigroup.*
(3) *For all* $a,b,c \in S$, $a\cdot(b+c) = a\cdot b + a\cdot c$ *and* $(a+b)\cdot c = a\cdot c + b\cdot c$.
(4) $0\cdot x = x\cdot 0 = 0 \ \forall \ x \in S$.

Definition 2.2. [4] *Let $(S,+,\cdot)$ be a semiring. $\preceq$ is said to be a Canonical Pre-order if for $a,b \in S$, $a \preceq b$ if and only if there exists an element $c \in S$ such that $a+c=b$.*

Definition 2.3. [6] *Let* $G = (V, E \subset V \times V)$ *be a given graph with* $V, E \neq \phi$. *For any semiring* $(S, +, \cdot)$, *a semiring-valued graph (or a* $S-$ *valued graph),* G^S, *is defined to be the graph* $G^S = (V, E, \sigma, \psi)$ *where* $\sigma : V \to S$ *and* $\psi : E \to S$ *are defined to be*

$$\psi(x,y) = \begin{cases} min\{\sigma(x), \sigma(y)\} & if\ \sigma(x) \preceq \sigma(y)\ or\ \sigma(y) \preceq \sigma(x) \\ 0 & otherwise \end{cases}$$

for every unordered pair (x, y) *of* $E \subset V \times V$. *We call* σ, *a* $S-$ *vertex set and* ψ, *a* $S-$ *edge set of* S*-valued graph* G^S.

Definition 2.4. [7]*An edge* e *in* G^S *is said to be a weight dominating edge if* $\psi(e_i) \preceq \psi(e)$ $\forall e_i \in N_S[e]$.

Definition 2.5. [7]*A subset* $D \subseteq E$ *is said to be a weight dominating edge set if for each* $e \in D$, $\psi(e_i) \preceq \psi(e)$, $\forall e_i \in N_S[e]$.

Definition 2.6. [3]*A set* $S \subseteq E$ *is a restrained dominating set if every edge* $E - S$ *is incident to an edge in* S *and another edge in* $E - S$.

Definition 2.7. [7]*A set* $M \subseteq E$ *is an independent edge set of* G^S *if* $f, g \in E$ *such that* $N_S(f) \cap (g, \psi(g)) = \phi$.

Definition 2.8. [7]*A subset* $M \subseteq E$ *is said to be a maximal independent edge set if*

(1) M *is an independent edge set.*
(2) *If there is no subset* M' *of* E *such that* $M \subset M' \subset E$ *and* M' *is an independent edge set.*

3. Restrained Edgs Domination on $S-$ Valued Graphs

In this section, we introduce the notion of restrained edge domination on $S-$valued graph, analogous to the notion in crisp graph theory, and prove some simple results.

Definition 3.1. *Consider the* $S-$ *valued graph* $G^S = (V, E, \sigma, \psi)$. *A subset* $D \subseteq E$ *is said to be a restrained weight dominating edge set if*

(1) D *is a weight dominating edge set.*
(2) *For each* $e \in E - D$ *is dominated by an edge in* D *and also by an edge in* $E - D$.

Example 3.2. Let $(S = \{0, a, b, c\}, +, \cdot)$ be a semiring with the following Cayley Tables:

$+$	0	a	b	c
0	0	a	b	c
a	a	a	a	a
b	b	a	b	b
c	c	a	b	c

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	b	b	b

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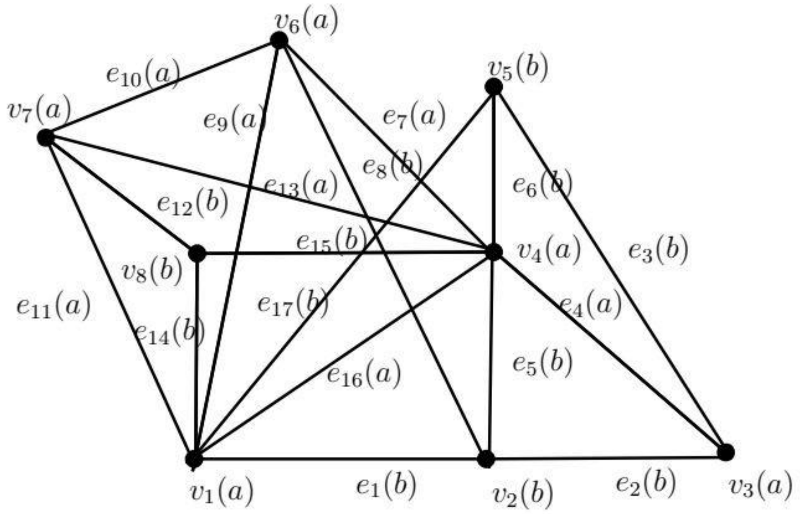
Let $\preceq$ be a canonical pre-order in S, given by

$$0 \preceq 0,\ 0 \preceq a,\ 0 \preceq b,\ 0 \preceq c,\ a \preceq a,\ b \preceq b,\ b \preceq a,\ c \preceq c,\ c \preceq a,\ c \preceq b$$

Consider the $S-$ graph G^S :



Define $\sigma : V \to S$ by

$$\sigma(v_1) = \sigma(v_3) = \sigma(v_4) = \sigma(v_6) = \sigma(v_7) = a, \sigma(v_2) = \sigma(v_5) = \sigma(v_8) = b$$

and $\psi : E \to S$ by
$\psi(e_1) = \psi(e_2) = \psi(e_3) = \psi(e_5) = \psi(e_6) = \psi(e_8) = \psi(e_{12}) = \psi(e_{14}) = b;$
$\psi(e_{15}) = \psi(e_{17}) = b;$
$\psi(e_4) = \psi(e_7) = \psi(e_9) = \psi(e_{10}) = \psi(e_{11}) = \psi(e_{13}) = \psi(e_{16}) = a$
Clearly $D = \{e_4, e_7, e_9, e_{10}, e_{11}, e_{13}, e_{16}\}$ is a restrained weight dominating edge set. Further $D_1 = \{e_4, e_7, e_9, e_{13}, e_{16}\}, D_2 = \{e_7, e_9, e_{10}, e_{11}, e_{13}, e_{16}\}$, $D_3 = \{e_4, e_7 e_{10}, e_{11}, e_{16}\}, D_4 = \{e_4, e_{10}, e_{16}\}, D_5 = \{e_4, e_9, e_{10}\}$ are all restrained weight dominating edge sets.

Definition 3.3. *A subset $D \subseteq E$ is said to be a minimal restrained weight dominating edge set if*

(1) *D is a restrained weight dominating edge set.*
(2) *No proper subset of D is a restrained weight dominating edge set.*

In example 3.2, $D_4 = \{e_4, e_{10}, e_{16}\}$ and $D_5 = \{e_4, e_9, e_{10}\}$ are minimal restrained weight dominating edge set.

Definition 3.4. *The restrained edge domination number of G^S denoted by $\gamma^S_{RE}(G^S)$ is defined by $\gamma^S_{RE}(G^S) = (|D|_S, |D|)$, where D is the minimal restrained weight dominating edge set.*

In example 3.2, $D_4 = \{e_4, e_{10}, e_{16}\}$ is a minimal restrained weight dominating edge set.

$$\gamma^S_{RE}(G^S) = (|D_4|_S, |D_4|) = (|D_5|_S, |D_5|) = (a, 3)$$

Definition 3.5. *A subset $D \subseteq E$ is said to be a maximal restrained weight dominating edge set if*

(1) *D is a restrained weight dominating edge set.*
(2) *If there is no subset D' of E such that $D \subset D' \subset E$ and D' is a restrained weight dominating edge set.*

In example 3.2, $D = \{e_4, e_7, e_9, e_{10}, e_{11}, e_{13}, e_{16}\}$ is a maximal restrained weight dominating edge set.

Definition 3.6. *A subset $M \subseteq E$ is said to be an independent restrained weight dominating edge set if M is both independent edge set and a restrained weight dominating edge set.*

In example 3.2, $D_5 = \{e_4, e_9, e_{10}\}$ is an independent restrained weight dominating edge set.

Theorem 3.7. *A restrained weight dominating edge set D of a graph G^S is a minimal restrained weight dominating edge set of G iff every edge $e \in D$ satisfies atleast one of the following properties:*

(1) *there exist an edge $f \in E - D$, such that $N_S(f) \cap (D \times S) = \{(e, \psi(e))\}$*
(2) *e is adjacent to no edge of D.*

Proof : *Let $e \in D$. Assume that e is adjacent to no edge of D, then $D - \{e\}$ cannot be a restrained weight dominating edge set. $\Rightarrow$ D is a minimal restrained weight dominating edge set.*
On the other hand, if for any $e \in D$ there exist a , $f \in E - D$ such that $N_S(f) \cap (D \times S) = \{(e, \psi(e))\}$
Then f is incident to $e \in D$ and no other edge of D.
In this case also, $D - \{e\}$ cannot be a restrained weight dominating edge set of G^S.
Hence D is a minimal restrained weight dominating edge set.
Conversely, *assume that D is a minimal restrained weight dominating edge set of G^S.*
Then for each $e \in D$, $D - \{e\}$ is not a restrained weight dominating edge set of G^S.
$\therefore$ there exist an edge, $f \in E - (D - \{e\})$ that is incident to no edge of $(D - \{e\})$.
If $f = e$, then e is incident to no edge of D.
If $f \neq e$ then D is a restrained weight dominating edge set and $f \notin D \Rightarrow f$ is incident to atleast one edge of D. However f is not incident to any edge of $D - \{e\}$.
$\Rightarrow$ $N_S(f) \cap D \times S = \{(e, \psi(e))\}$.

Theorem 3.8. *A set $D \subseteq E$ of G^S is an independent restrained weight dominating edge set iff D is a maximal independent edge set in G^S.*
Proof: *Clearly every maximal independent edge set D in G^S is an independent restrained weight dominating edge set.*
Conversely, *assume that D is an independent restrained weight dominating edge set.*

Then D *is independent and every edge not in* D *is incident to an edge of* D *and therefore* D *is a maximal independent edge set in* G^S.

Theorem 3.9. *Every maximal independent edge set of edges* D *in* G^S *is a minimal restrained weight dominating edge set.*
Proof : *Let* D *be a maximal independent edge set of edges* D *in* G^S. *Then by theorem* 3.8 , D *is a restrained weight dominating edge set.*
Since D *is independent, every edge of* D *is incident to no edge of* D.
Thus, every edge of D *satisfies the second condition of theorem* 3.7. *Hence* D *is a minimal restrained weight dominating edge set in* G^S.

Theorem 3.10. *If* $D \subseteq E$ *is a minimal restrained weight dominating edge set of* G^S *without* $S-$ *isolated edges then* $E - D$ *is also a restrained weight dominating edge set of* G^S.
Proof: *Let* $e \in D$. *Then by theorem* 3.7,

(1) *there exist an edge* $u \in E - D$ *such that* $N_S(u) \cap D = \{e\}$
(2) e *is incident to no edge of* D.

In the first case, e *is incident to some edge in* $E - D$.
In the second case, e *is an* $S-$ *isolated edge of the subgraph spanned by* $\langle D \rangle$.
But e *is not* $S-$ *isolated in* G^S.
Hence e *is incident to some edge of* $E - D$.
Thus $E - D$ *is a restrained weight dominating edge set of* G^S.

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