RESTRAINED VERTEX DOMINATION ON $S-$ VALUED GRAPHS

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ABSTRACT. In this paper, we introduce the notion of restrained vertex domination on $S-$ valued graphs and study some properties.

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1. INTRODUCTION

In[6], the authors introduced the notion of $S-$ valued graphs, where S is a semiring. In graph theory, domination of graphs is the most powerful area of research for, it has several applications in other areas of sciences. It was initicted by Berge [1]. In [7], the authors have studied the vertex domination on $S-$ valued graphs. In this paper we discuss the notion of restrained vertex domination on $S-$ valued graphs.

2. PRELIMINARIES

In this section we recall some basic definitions that are needed for our work.

Definition 2.1. [4] *A semiring $(S, +, \cdot)$ is an algebraic system with a non-empty set S together with two binary operations $+$ and $\cdot$ such that*

(1) $(S, +, 0)$ is a monoid.
(2) $(S, \cdot)$ is a semigroup.
(3) For all $a, b, c \in S$, $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$.
(4) $0 \cdot x = x \cdot 0 = 0 \;\forall\; x \in S$.

Definition 2.2. [4] *Let $(S, +, \cdot)$ be a semiring. $\preceq$ is said to be a Canonical Pre-order if for $a, b \in S$, $a \preceq b$ if and only if there exists an element $c \in S$ such that $a + c = b$.*

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Definition 2.3. [6] *Let $G = (V, E \subset V \times V)$ be a given graph with $V, E \neq \phi$. For any semiring $(S, +, \cdot)$, a semiring-valued graph (or a $S-$ valued graph), G^S, is defined to be the graph $G^S = (V, E, \sigma, \psi)$ where $\sigma : V \to S$ and $\psi : E \to S$ are defined to be*

$$\psi(x, y) = \begin{cases} \min\{\sigma(x), \sigma(y)\} & \text{if } \sigma(x) \preceq \sigma(y) \text{ or } \sigma(y) \preceq \sigma(x) \\ 0 & \text{otherwise} \end{cases}$$

for every unordered pair (x, y) of $E \subset V \times V$. We call σ, a $S-$vertex set and ψ, a $S-$ edge set of S-valued graph G^S.

Definition 2.4. [7]*A vertex v in G^S is said to be a weight dominating vertex if $\sigma(u) \preceq \sigma(v)$, $\forall\ u \in N_S[v]$.*

Definition 2.5. [7]*A subset $D \subseteq V$ is said to be a weight dominating vertex set if for each $v \in D$, $\sigma(u) \preceq \sigma(v)$, $\forall\ u \in N_S[v]$.*

Definition 2.6. [3]*A set $S \subseteq V$ is a restrained dominating set if every vertex $V - S$ is adjacent to a vertex in S and another vertex in $V - S$.*

Definition 2.7. [7]*A subset $D \subseteq V$ is an independent vertex set of G^S if $u, v \in D$ such that $N_S(u) \cap (v, \sigma(v)) = \phi$.*

Definition 2.8. [7]*A subset $D \subseteq V$ is said to be a maximal independent vertex set if*

(1) D is an independent vertex set.
(2) If there is no subset D' of V such that $D \subset D' \subset V$ and D' is an independent vertex set.

3. Restrained Vertex Domination on $S-$ Valued Graphs

In this section, we introduce the notion of restrained vertex domination on $S-$ valued graph, analogous to the notion in crisp graph theory, and prove some simple results.

Definition 3.1. *Consider the $S-$ valued graph $G^S = (V, E, \sigma, \psi)$. A subset $D \subseteq V$ is said to be a restrained weight dominating vertex set if*

(1) D is a weight dominating vertex set.
(2) For each $v \in V - D$ is dominated by a vertex in D and also by a vertex in $V - D$.

Example 3.2. Let $(S = \{0, a, b, c\}, +, \cdot)$ be a semiring with the following Cayley Tables:

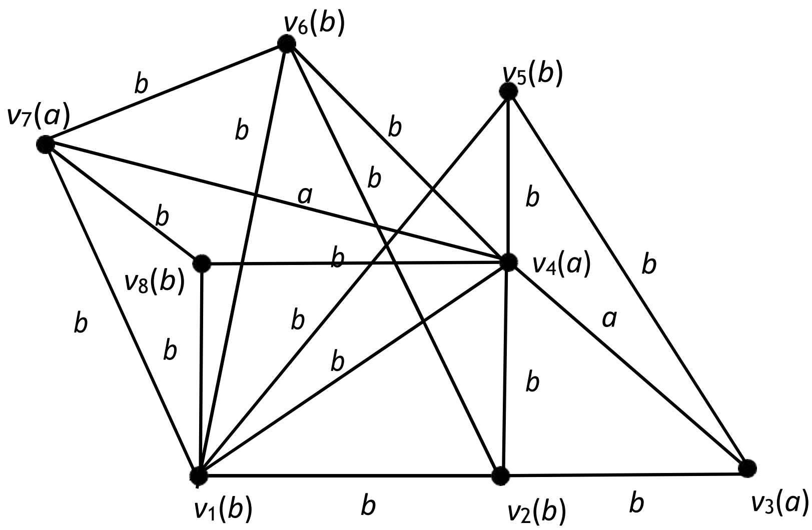
+	0	*a*	*b*	*c*
0	0	*a*	*b*	*c*
a	*a*	*a*	*a*	*a*
b	*b*	*a*	*b*	*b*
c	*c*	*a*	*b*	*c*

·	0	*a*	*b*	*c*
0	0	0	0	0
a	0	*a*	*a*	*a*
b	0	*b*	*b*	*b*
c	0	*b*	*b*	*b*

Let $\preceq$ be a canonical pre-order in S, given by

$$0 \preceq 0,\ 0 \preceq a,\ 0 \preceq b,\ 0 \preceq c,\ a \preceq a,\ b \preceq b,\ b \preceq a,\ c \preceq c,\ c \preceq a,\ c \preceq b$$

Consider the $S-$ graph G^S :



Define $\sigma : V \to S$ by

$$\sigma(v_1) = \sigma(v_2) = \sigma(v_5) = \sigma(v_6) = \sigma(v_8) = b,\ \sigma(v_3) = \sigma(v_4) = \sigma(v_7) = a$$

and $\psi : E \to S$ by

$\psi(v_1, v_2) = \psi(v_1, v_4) = \psi(v_1, v_5) = \psi(v_1, v_6) = \psi(v_1, v_7) = \psi(v_1, v_8) = \psi(v_2, v_3) = \psi(v_2, v_4) = \psi(v_2, v_6) = \psi(v_3, v_5) = \psi(v_4, v_5) = \psi(v_4, v_6) = \psi(v_4, v_8) = \psi(v_6, v_7) = \psi(v_7, v_8) = b,$

$\psi(v_3, v_4) = \psi(v_4, v_7) = a.$

Clearly $D = \{v_3, v_4, v_7\}$ is a restrained weight dominating vertex set.

Further $D_1 = \{v_3, v_4\}$, $D_2 = \{v_3, v_7\}$, $D_3 = \{v_4, v_7\}$, $D_4 = \{v_4\}$ are all restrained weight dominating vertex sets.

Definition 3.3. *A subset $D \subseteq V$ is said to be a minimal restrained weight dominating vertex set if*

(1) D is a restrained weight dominating vertex set.

(2) No proper subset of D is a restrained weight dominating vertex set.

In example 3.2, $D_4 = \{v_4\}$ is a minimal restrained weight dominating vertex set.

Definition 3.4. *The restrained vertex domination number of G^S denoted by $\gamma^S_{RV}(G^S)$ is defined by $\gamma^S_{RV}(G^S) = (|D|_S, |D|)$, where D is the minimal restrained weight dominating vertex set.*

In example 3.2, $D_4 = \{v_4\}$ is a minimal restrained weight dominating vertex set.

$$\gamma^S_{RV}(G^S) = (|D_4|_S, |D_4|) = (a, 1)$$

Definition 3.5. *A subset $D \subseteq V$ is said to be a maximal restrained weight dominating vertex set if*

(1) D is a restrained weight dominating vertex set.
(2) If there is no subset D' of V such that $D \subset D' \subset V$ and D' is a restrained weight dominating vertex set.

In example 3.2, $D = \{v_3, v_4, v_7\}$ is a maximal restrained weight dominating vertex set.

Definition 3.6. *A subset $M \subseteq V$ is said to be an independent restrained weight dominating vertex set if M is both independent vertex set and a restrained weight dominating vertex set.*

In example 3.2, $D_2 = \{v_3, v_7\}$ is an independent restrained weight dominating vertex set.

Theorem 3.7. *A restrained weight dominating vertex set D of a graph G^S is a minimal restrained weight dominating vertex set of G iff every vertex $v \in D$ satisfies atleast one of the following properties:*

(1) *there exist a vertex $f \in V - D$, such that $N_S(f) \cap (D \times S) = \{(v, \sigma(v))\}$*
(2) v is adjacent to no vertex of D.

Proof : *Let $v \in D$. Assume that v is adjacent to no vertex of D, then $D - \{v\}$ cannot be a restrained weight dominating vertex set. $\Rightarrow$ D is a minimal restrained weight dominating vertex set.*
On the other hand, if for any $v \in D$ there exist a $f \in V - D$ such that $N_S(f) \cap (D \times S) = \{(v, \sigma(v))\}$
Then f is adjacent to $v \in D$ and no other vertex of D.
In this case also, $D - \{v\}$ cannot be a restrained weight dominating vertex set of G^S.
Hence D is a minimal restrained weight dominating vertex set.
Conversely, *assume that D is a minimal restrained weight dominating vertex set of G^S.*
Then for each $v \in D$, $D - \{v\}$ is not a restrained weight dominating vertex set of G^S.
$\therefore$ *there exist a vertex, $f \in V - (D - \{v\})$ that is adjacent to no vertex of $(D - \{v\})$.*
If $f = v$, then v is adjacent to no vertex of D.
If $f /= v$, then D is a restrained weight dominating vertex set and $f \notin D \Rightarrow f$ is adjacent to atleast one vertex of D. However f is not adjacent to any vertex of $D - \{v\}$.
$\Rightarrow N_S(f) \cap D \times S = \{(v, \sigma(v))\}$.

Theorem 3.8. *A set $D \subseteq V$ of G^S is an independent restrained weight dominating vertex set iff D is a maximal independent vertex set in G^S.*
Proof: *Clearly every maximal independent vertex set D in G^S is an independent restrained weight dominating vertex set.*
Conversely, *assume that D is an independent restrained weight dominating vertex*

set.
Then D is independent and every vertex not in D is adjacent to a vertex of D and therefore D is a maximal independent edge set in G^S.

Theorem 3.9. *Every maximal independent vertex set of vertices D in G^S is a minimal restrained weight dominating vertex set.*
Proof : *Let D be a maximal independent vertex set of vertices D in G^S. Then by theorem 3.8, D is a restrained weight dominating vertex set.*
Since D is independent, every vertex of D is adjacent to no vertex of D.
Thus, every vertex of D satisfies the second condition of theorem 3.7. Hence D is a minimal restrained weight dominating vertex set in G^S.

Theorem 3.10. *If $D \subseteq V$ is a minimal restrained weight dominating vertex set of G^S without $S-$ isolated vertices then $V - D$ is also a restrained weight dominating vertex set of G^S.*
Proof: *Let $v \in D$. Then by theorem 3.7,*

(1) *there exist a vertex $u \in V - D$ such that $N_S(u) \cap D = \{v\}$*
(2) v is adjacent to no vertex of D.

In the first case, v is adjacent to some vertex in $V - D$.
In the second case, v is an $S-$ isolated vertex of the subgraph spanned by $\langle D \rangle$.
But v is not $S-$ isolated in G^S.
Hence v is adjacent to some vertex of $V - D$.
Thus $V - D$ is a restrained weight dominating vertex set of G^S.

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